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Ordering, pricing and allocation in a service supply chain

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ABSTRACT

In this paper, we study a two-stage game problem on pricing, ordering and allocation in a service supply chain, where one supplier sells a product with a fixed capacity to customers via two retailers under wholesale price contracts. The two retailers face random demands and order from the supplier. The supplier needs to allocate its capacity to retailers according to some allocation rule when its capacity cannot fit the retailers' order. We study two decentralized supply chains, where retail prices are determined by the supplier or the retailers. For each model, we derive and characterize the equilibrium by transforming the game problem into an optimization problem. We find that under the leader of the supplier the competition between the two retailers is eliminated and each retailer just orders its optimal quantity. So, the retailers' behavior in the game is not influenced by the supplier's allocation rule. Furthermore, with pricing power, the supplier can get higher profit but the retailers would not necessarily.

1. Introduction

Today, there are often many channels for a service firm to sell its product such as flight tickets or rooms in a hotel. For example, airlines often sell their tickets through different retailers, including traditional travel agents, online travel agents, and airlines' own direct retailing sites. In Chinese air market of 2008, the market share of traditional travel agents is about 75%, that of online travel agents is about 15%, and that of airlines' own direct retailing sites is lower than 10%. The ratio of tickets sold by agents is over 90%. Moreover, the market share of online travel agents is increasing during these years. There are many, national or local, traditional travel agents such as CITS and Chunqiu and online agents such as Ctrip, eLong and Taobao in China.

The airline and its agents form a service supply chain (Hasija et al., 2008; Hu et al., 2010). The agents, especially those with a large amount of demand, like retailers order capacity from the airline (i.e., the supplier). A service firm in service industries often has fixed capacity. For example, a flight has a fixed number of seats and a hotel has a fixed number of rooms. Thus, the supplier needs to allocate its finite capacity among multiple retailers when the total order quantity from retailers exceeds its capacity. At the same time, there is a pricing problem in each retailer since different retailer may face different market. Hence, it is interesting to study the problem that how does a service firm sell its product/service to multiple retailers, where there occur game problems among the supplier and the retailers on ordering, pricing and allocation.

Armed with these insights, we study in this paper a two-stage game problem on ordering, pricing and allocation in a decentralized service supply chain consisting of one supplier and two retailers. The supplier with a fixed capacity sells a same product or provide a same service to customers through the two retailers, who compete in the retail market. Each retailer faces random demand which depends on retail prices at both retailers. This game problem consists of two stages. In the first stage, the supplier needs to determine a wholesale price and a retail price to each retailer. In the second stage, the two retailers compete the supplier's capacity, i.e., each retailer orders a quantity from the supplier. When the sum of order quantities from the two retailers exceeds the supplier's capacity, the capacity is allocated to the two retailers according to some allocation rule. We also study another type of decentralized supply chain which is similar to the previous one except that each retailer determines its retail price together with its order quantity in the second stage.

We show existence of equilibria for the two decentralized supply chains and characterize the equilibria. Based on the equilibria, we show that the competition between the two retailers is eliminated and each retailer just orders its optimal quantity under the leader of the supplier. So, the retailers' ordering behavior is independent of the supplier's allocation rule, which implies that the supplier can use a simple linear allocation rule to allocate its fixed capacity. Moreover, we show that with the power of determining retail prices the supplier can obtain higher profit but the retailers would not necessarily.

There are several directions in the literature concerning the problem studied in this paper. The first direction is revenue management with research areas of capacity allocation (or leg/class and origin/destination control, see e.g., McGill and van Ryzin,

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1999; Dai et al., 2006) and dynamic pricing (see e.g., Gallego and van Ryzin, 1994; Feng and Xiao, 2000). A comprehensive review of this literature can be found in the book Talluri and van Ryzin (2004). In this research, there is only one decision maker (the firm) who maximizes the revenue of the total system.

Recently, the research in revenue management is extended to a competitive environment and some types of games are involved. Regarding gaming in the revenue management literature, Netessine and Shumsky (2005) studied seat-inventory control with fixed prices for two airlines forming an alliance, and they proposed a revenue-sharing contract to coordinate the two (horizontal or vertical) airlines. Jerath et al. (2010) studied competition between two retailers using a two-period model.

The second direction is joint ordering and pricing. This topic is often studied in the framework of traditional supply chain, where the supplier's capacity is infinite. For example, Bernstein and Federgruen (2003) designed coordination mechanisms for a supply chain with one supplier and multi-retailers, in which retail competition is characterized by either a Bertrand model or a Cournot model. Bernstein and Federgruen (2005) further studied the case where individual demand is stochastic and depends on either all retailers' prices or only his own price. They characterized retailers' Nash equilibria and proved that the supply chain can be coordinated. The supplier's capacity is assumed to be infinite and there is no capacity allocation issue in the above papers.

Capacity choice and allocation problem were studied in another framework of supply chain. Here, the supplier needs to determine its capacity and then allocate it to its retailers. So, the capacity choice and allocation interact with retailers' decisions such as retail price and ordering quantity, for example, see Cachon and Lariviere (1999a, 1999b).

The most related work to this paper is Dai et al. (2005), who studied pricing competition among multiple retailers. Each retailer has finite capacity. They gave a condition for ensuring the existence of Nash equilibrium. In this paper, we study competition in a supply chain with one supplier and two retailers on decisions of ordering and pricing. Our problem here differs from Dai et al. (2005) in that (1) we study the problem in the framework of supply chains; (2) here each retailer's capacity is endogenous depending on both retailers' order quantities and the supplier's allocation rule; and (3) retail price at each retailer can be determined by either the supplier or the retailer itself.

Hu et al. (2010) is another work related to our paper. They considered competition in a supply chain, where the supplier can sell its product by itself or through the retailer. However, they focused on dynamic pricing in the framework of revenue management. While in this paper, we focus on a two-stage game problem on ordering, pricing and allocation in a supply chain where the supplier sells its products thorough two retailers.

The rest of the paper is organized as follows. In the next section, we give the models of two decentralized supply chains and of the centralized supply chain. In Sections 3 and 4 we study, respectively, the two decentralized supply chains. In Section 5 we give a comparison of the two models and with the centralized supply chain and give some numerical analysis. Some technical analysis for the two decentralized supply chain models are given in Section 6. Section 7 is a concluding section.

2. Models

We consider a service supply chain with one supplier and two retailers, denoted by S and R_1, R_2, respectively. The supplier sells seats with fixed capacity N to customers through two retailers under wholesale price contracts. The supplier first determines wholesale prices w_1 and w_2 for two retailers. Then, retailer i orders a quantity y_i from the supplier with a wholesale price w_i. We require that $y_i \le N$, $i=1,2$. It is not meaningful to order over the supplier's capacity. However, the total order quantity y_1+y_2 may be larger than N, i.e., $y_1+y_2>N$. In this case, the supplier should allocate its capacity N to the two retailers according to some allocation rule. We use $\{G_1(y_1,y_2), G_2(y_1,y_2)\}$ to represent the allocation rule. It means that when retailers 1 and 2 order y_1 and y_2 seats, respectively, the supplier will allocate $G_i(y_1,y_2)$ to retailer i. As an allocation rule, we require that $G_i(y_1,y_2)$ is increasing in y_i and decreasing in $y_j, j\neq i$, $G_i(y_1,y_2)\in[0,y_i]$, $G_i(y_1,y_2)=y_i$ when $y_1+y_2\le N$, and $G_1(y_1,y_2)+G_2(y_1,y_2)=N$ when $y_1+y_2\ge N$, for $i=1,2$. This requirement is certainly reasonable. A special case is the linear allocation rule in which retailer i is allocated $N\cdot y_i/(y_1+y_2)$ when $y_1+y_2>N$, otherwise its own order y_i, for $i=1,2$. We write $G_i(y_1,y_2)$ by simply G_i when no confusion occurs. We first assume that the allocation rule is predetermined and finally show that our results are true irrespectively of the allocation rule.

The demand faced by retailer i is a random variable $D_i(p_1,p_2)$ when the prices at retailer 1 and retailer 2 are p_1 and p_2, respectively. This means that the two retailers are competitive in the market. Let $F_i(\cdot|p_1,p_2)$ and $f_i(\cdot|p_1,p_2)$ be the distribution function (d.f.) and probability density function (p.d.f.) of $D_i(p_1,p_2)$, respectively, for $i=1,2$. We assume that both F_1 and F_2 are increasing general failure rate (IGFR), that is, the general failure rate $xf_i(x|p_1,p_2)/[1-F_i(x|p_1,p_2)]$ is strictly increasing in x for given p_1,p_2. The IGFR is often assumed in the literature of supply chain management, e.g., in Larivaiere and Porteus (2001), who show that the IGFR captures most common distributions such as the normal, uniform, and the majority of Gamma and Weibull.

In service industries such as airlines and hotels, there is little operation costs compared with firms' fixed sunk costs. So, we assume that the operation costs for the supplier and retailers are all zero.

As for determining retail prices, we consider two cases where both retail prices are determined by the supplier or by the two retailers, respectively. Hence, we will study two decentralized supply chains. As the benchmark, we also study the centralized supply chain. Therefore, we have three types of the supply chains, as follows.

Decentralized Model I: In this decentralized model, the supplier first determines wholesale prices (w_1,w_2) and retail prices (p_1,p_2) for both retailers. Then the two retailers independently make their order quantities y_1 and y_2, after which the supplier allocate its capacity to the two retailers, and finally the two retailers sell seats to customers. It is assumed that the supplier and retailers are risk-neutral, and so they want to maximize their own expected profits. Hence, the supplier and retailers face a Stackelberg game. The supplier decides w_1,w_2,p_1,p_2 to maximize its expected profit, as described by

$$\max_{w_1,w_2,p_1,p_2} \pi_S = w_1G_1(y_1^e,y_2^e)+w_2G_2(y_1^e,y_2^e), \tag{1}$$

where (y_1^e,y_2^e) is the Nash equilibrium of the following sub-game faced by the two retailers (here we define $a\wedge b=\min(a,b)$ for real numbers a,b)

$$\max_{y_i\le N} \pi_i(y_1,y_2)=p_i(D_i(p_1,p_2)\wedge G_i(y_1,y_2))-w_iG_i(y_1,y_2), \tag{2}$$

for $i=1,2$, with given (w_1,w_2,p_1,p_2). So, after the supplier determining wholesale prices and retail prices, the two retailers compete the supplier's fixed capacity via their ordering quantities, as described in Eq. (2). We call this sub-game as the *ordering game*.

Decentralized Model II: In this decentralized model, the supplier first determines wholesale prices w_1 and w_2 for retailers 1 and 2, and then retailer i determines its order quantity y_i and