Int. J. Production Economics 144 (2013) 610–617

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Optimal production lot with imperfect production process under permissible delay in payments and complete backlogging

Liang-Yuh Ouyang [a], Chun-Tao Chang [b,*]

[a] Department of Management Sciences, Tamkang University, Tamsui, Taipei 25137, Taiwan, ROC
[b] Department of Statistics, Tamkang University, Tamsui, Taipei 25137, Taiwan, ROC

ARTICLE INFO

Article history:
Received 27 March 2012
Accepted 10 April 2013
Available online 25 April 2013

Keywords:
Inventory
Imperfect production process
Complete backlogging
Permissible delay in payments
Arithmetic–geometric mean inequality

ABSTRACT

The traditional economic production quantity (EPQ) model assumes that the production products are all perfect. It is not always true in the real production system, due to imperfect production process or other factors, imperfect quality items may be produced. Furthermore, it is well-known that the total production-inventory costs can be reduced by reworking the imperfect quality items produced with a relatively smaller additional reworking and holding costs. In addition, the permissible delay in payments offered by the supplier is widely adopted in the practical business market. In this study, we explore the effects of the reworking imperfect quality items and trade credit on the EPQ model with imperfect production processes and complete backlogging. A mathematical model which includes the reworking and shortage costs, interest earned and interest charged is presented. Besides, an arithmetic-geometric mean inequality approach is employed and an algorithm is developed to find the optimal production policy. Furthermore, some numerical examples and sensitivity analysis are provided to demonstrate the proposed model.

1. Introduction

In the classical EPQ model, it is implicitly assumed that the production products are all perfect. However, in the real manufacturing circumstance, due to imperfect production process or other factors, the defective items may be produced. Several scholars have developed various analytical models to study the EPQ model with defective items. Rosenblatt and Lee (1986) were one of the early researchers who studied the effects of an imperfect production process on the optimal production cycle time for the classical economic manufacturing quantity (EMQ) model. Porteus (1986) introduced a relationship between process quality control and lot sizing. Zhang and Gerchak (1990) presented joint lot sizing and inspection policy in an economic order quantity (EOQ) model with random yield. Cheng (1991) developed an EOQ model with demand-dependent unit production cost and imperfect production processes. Ben-Daya (2002) formulated an integrated model with joint determination of EPQ and preventive maintenance level under an imperfect process. Lin et al. (2003) examined an integrated production-inventory model for imperfect production processes under inspection schedules. Recently, Sana (2010) developed a production-inventory model in an imperfect production process. Many related articles in EPQ models with imperfect quality items can be found such as Salameh and Jaber (2000), Sana et al. (2007), Yoo et al. (2009), Sana (2011), Sarker et al. (2010), Sarker and Moon (2011), Sarker (2012), Yoo et al. (2012) and their references.

All of the above studies about EPQ models with imperfect production processes focused on determining the optimal production lot size. The issue that the imperfect quality items can be reworked was ignored. It is well-known that the total production-inventory costs can be reduced by reworking the imperfect quality items produced with a relatively smaller additional reworking and holding costs. Numerous studies on the problems of EPQ model with rework process have been discussed by Liu and Yang (1996), Hayek and Salameh (2001), Chiu (2003, 2008), Chiu et al. (2004, 2010), Jamal et al. (2004), Chiu and Chiu (2006), Taleizadeh et al. (2010) and their references.

Actually, today trade credit is widespread and represents an important proportion of company finance. Businesses, especially small businesses, with limited financing opportunities, may be financed by their suppliers rather than by financial institutions (Petersen and Rajan, 1997). On the other hand, offering trade credit to retailers may encourage the supplier sales and reduce the on-hand stock level (Emery, 1987). Goyal (1985) was the first to establish an EOQ model with a constant demand rate under the condition of a permissible delay in payments. Teng (2002) modified Goyal's (1985) model by considering the difference between the selling price and purchase cost, and found that the economic replenishment interval and order quantity decrease under the

* Corresponding author. Tel.: +886 2 26215656x2632; fax: +886 2 26209732.
E-mail address: chuntao@stat.tku.edu.tw (C.-T. Chang).

0925-5273/$ - see front matter © 2013 Published by Elsevier B.V.
http://dx.doi.org/10.1016/j.ijpe.2013.04.027

permissible delay in payments in certain cases. Chang et al. (2003) developed an EOQ model with deteriorating items under supplier's credits linked to ordering quantity. Tsao et al. (2011) proposed a production model with reworking imperfect items and trade credit. Numerous interesting and relevant paper related to trade credits such as Aggarwal and Jaggi (1995), Jamal et al. (1997), Chang (2004), Ouyang et al. (2005), Teng et al. (2005), Goyal et al. (2007), Liao (2008), Teng and Chang (2009), Chang et al. (2010), Musa and Sani (2012), Su (2012) and so on.

Because trade credit is a widespread and popular payment method, thus, in order to respond to the real business behavior, we study an EPQ model with imperfect quality and complete backlogging when the supplier offers a permissible delay in payments. Some simple algebraic manipulations and an arithmetic–geometric mean inequality approach are employed to determine the optimal production lot size and backorder level. Besides, an algorithm is developed to find the optimal solution. Finally, some numerical examples and sensitivity analysis are presented to illustrate the proposed model.

2. Notation and assumptions

The following notation and assumptions will be adopted in this article.
Notation

P	production rate
λ	demand rate
K	setup cost for each production run
v	purchasing cost of raw material per unit
c	production cost per item including purchasing cost and inspecting cost, $c>v$
s	selling price per unit, $s>c$
h	holding cost per item per unit time, excluding the interest charge
b	shortage cost per item per unit time
x	the proportion of imperfect quality items produced, where $0<x<1$
d	the production rate of imperfect quality regular production process per unit time, where $d=Px$
P_1	the rate of reworking of imperfect quality items
c_R	reworking cost for each imperfect quality item
Q	production lot size for each cycle
B	allowable backorder level
T	production cycle length
H_1	maximum level of on-hand inventory when regular production process stops
H	maximum level of on-hand inventory in units, when the reworking ends
M	permissible delay period offered by the supplier
I_c	the interest charged per dollar per unit time in stocks by the supplier
I_e	the interest earned per dollar per unit time
$TC_i(Q, B)$	inventory total cost per cycle for case i, $i=1, 2, 3$
$TCU_i(Q,B)$	inventory total cost per unit time for case i, i.e., $TCU_i(Q, B)=TC_i(Q,B)/T$, $i=1, 2, 3$.

Assumptions

(1) Each product is made by a raw material.
(2) Production rate for perfect items is larger than demand rate, i.e., $(1-x)P>\lambda$.
(3) All imperfect quality items can be reworked and become perfect items.
(4) Shortages are allowed and completely backlogged.
(5) The supplier provides the manufacturer a permissible delay in payments. During the trade credit period the account is not settled, generated sales revenue is deposited in an interest bearing account with interest rate I_e. At the end of the permissible delay, the manufacturer pays off all units ordered, and starts paying for the interest charges on the raw material in stocks with interest rate I_c.

3. Mathematical formulation

First, a short problem description is provided. The manufacturer buys all raw materials Q units per order from the supplier to product and the unit purchasing price of raw material is v. The supplier offers the manufacturer a permissible delay period M. That is, the manufacturer buys raw materials at time zero and must pay the purchasing cost vQ at time M. The unit production cost is c and the unit selling price of the perfect items is s. The production process starts at time zero. A constant product rate P is considered during the regular production uptime. The process may generate x percent of imperfect quality items at a production rate $d=Px$. Thus, the produced items fall into two groups, the perfect and the imperfect. The production rate for perfect item $(1-x)P$ is larger than the demand rate λ. All imperfect quality items are assumed to be reworkable at a rate of P_1, and rework process starts when regular production process ends. All reworked items are assumed to become perfect items after rework process. Shortages are allowed and completely backlogged. In this situation, the production-inventory system follows the pattern depicted in Fig. 1. From Fig. 1, the expressions of production uptime t_1 and t_2, reworking time t_3, production downtime t_4, shortage permitted time t_5, the maximum levels of on-hand inventory H_1 and H, and the cycle length T are as follows:

$$t_1=\frac{B}{P-d-\lambda} \tag{1}$$

$$t_2=\frac{H_1}{P-d-\lambda} \tag{2}$$

$$t_3=\frac{xQ}{P_1}=\frac{dQ}{P_1 P} \tag{3}$$

$$t_4=\frac{H}{\lambda} \tag{4}$$

$$t_5=\frac{B}{\lambda} \tag{5}$$

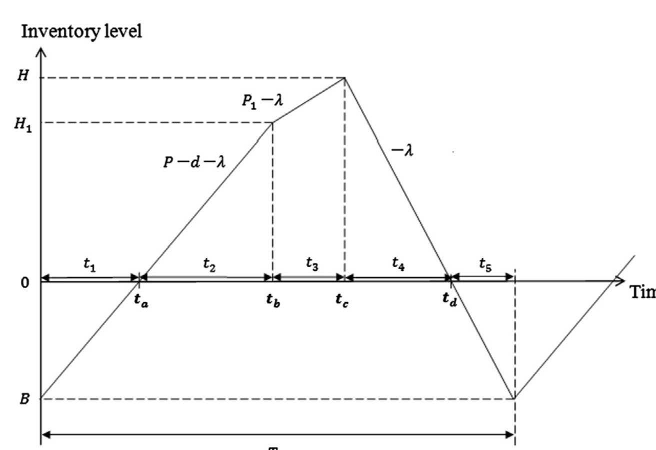

Fig. 1. Graphical representation of the inventory system.