Int. J. Production Economics 143 (2013) 41–52

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

A new method for robustness in rolling horizon planning

D. Bredström, P. Flisberg, M. Rönnqvist *

Norwegian School of Economics and Business Administration, Bergen, Norway

ARTICLE INFO

Article history:
Received 10 January 2010
Accepted 7 February 2011
Available online 22 February 2011

Keywords:
Robust optimization
Uncertainty
Transportation
Distribution
Production planning

ABSTRACT

In this paper, we describe a new method to solve Linear Programming (LP) problems which have uncertain right-hand-sides. We apply this to planning problems where a rolling planning horizon is used and where robustness is important. In particular, we are interested in applications where the uncertainty has an underlying structure and can be described with practical constraints. The method proposed is based on a decomposition scheme where we iteratively solve an upper level problem for the first time period in which the parameters are assumed to be known. The lower level problem uses the upper level solution and computes a worst case scenario for an anticipation period that has uncertain parameters. Information about how the worst case scenario is affected by the upper level decisions is given back as a valid inequality. This process is repeated until the upper level solution satisfies the last generated valid inequality. The models used in the solution process can be kept as small as the corresponding deterministic model which has no uncertainties. We test the proposed method on an integrated production, transportation and inventory planning problem. We make use of simulations to compare our approach with a traditional deterministic approach with safety stocks. The result shows that the proposed method works well and performs better than the deterministic approach.

1. Introduction

The interest for solving industrial problems where some data, e.g. demand forecast, is unknown is increasing. One reason for this is the daily use of advanced planning systems in Enterprise Resource Planning (ERP) systems in which planners need to consider uncertainty in data. There are several levels of planning horizons in such systems, ranging from long term strategic planning over several years down to short-term operative planing on a daily basis. In this article, we focus on planning problems in medium-term so-called tactical planning problems. Typical problems of this kind are integrated production, transportation and inventory planning problems where the planning period ranges from a few months to one year. The decisions that need to be made are how much to produce, how much to keep in inventory and how to transport products between production units to customers. These problems are often formulated as Linear Programming (LP) problems. A typical and well used planning approach in ERP systems is to use rolling horizon planning. In tactical planning, the planning horizon, say one year, is divided into time periods of for example 12 months. For each time period, we need information about production, transportation and storage capacities and demand. In addition, we need information about transportation and inventory costs. Once we have solved the planning problem, we only implement the decisions found for the first period as the situation will change in the remaining periods. During this period we may use tactical decisions and use other operational models, for example, routing and production planning models, to make the short-term operational decisions. Towards the end of the period, we go back and solve the tactical model which now covers the next 12 months or the remaining months of the year. The latter situation would be the case of for example in annual budget planning. Any information collected during the month is used to update the model.

One example of the planning problem discussed above is the annual planning of raw material for heating plants. Heating plants are situated in towns and produce hot water which is transported out to houses and apartments through a pipe system and which is used for heating. Many such plants are in use and many are being built and planned, for example in Sweden. The demand is expressed in energy content, i.e./MWh, at each heating plant. One difficult aspect of the planning is that the demand at the heating plants is dependent on the weather, i.e. the average temperatures. For example, the demand during a cold Swedish winter month may be several times larger than during a warm summer month. Moreover, the temperature is very uncertain and has some special characteristics, e.g. the temperature can be very cold or warm during some short time periods. This leads to large

* Corresponding author.
E-mail address: mikael.ronnqvist@nhh.no (M. Rönnqvist).

0925-5273/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2011.02.008

variations in the demand for a specific time period of, say, one month. Further, the average temperature over a long term is relatively constant. Therefore, the annual contract between the energy provider and each heating plant expresses a flexible demand per month but a fixed annual energy volume. A typical contract stipulates a demand level for each month for one year, typically July–June. The demand for the next month is fixed, but the heating plant can increase or decrease the demand level for the following months on a given monthly date in a rolling horizon planning by, say, 10%. However, the agreed total annual energy volume must be maintained. This implies that the total deviation in demand over all time periods is bound to zero. This provides the heating plants with the possibility to compensate for the weather but creates a planning problem for the bio-fuel company that must ensure a robust plan in which all possible scenarios are considered.

We now turn back to our general integrated production, transportation and inventory planning problem. It is important to find a robust plan which can be defined as being capable of performing well in uncertain future conditions, such as variations in demand. A traditional solution approach is to formulate a deterministic model with safety stock levels at for example terminals or distribution centers (DC). The purpose of the safety stock is to take into account uncertainty in demand. The main problems are to decide the level of the safety stock and the distribution of an entire stock over several terminals. An advantage is that the size of the planning problem can be kept small, as uncertainty is handled through safety stock levels. Another approach is to use stochastic optimization. However, such models require full knowledge of the distributions of the uncertain data and such information is rarely available in practice. In addition, such models are generally very large and difficult to solve, even for small instances. The challenges presented by stochastic, have made robust optimization to receive a lot of attention. In robust optimization the uncertainty of the parameters is modeled as lower and upper bounds without any need for exact distributions. The classical approach to robust optimization is to search for an optimal solution which has the property that the solution will satisfy all possible outcomes. This is called a static robust solution. The objective is typically to minimize the worst case cost. Early work was done by Soyster (1973), who proposed a worst case model for linear optimization such that constraints are satisfied under all possible perturbations of the uncertain data of the underlying model. Kouvelis and Yu (1997) describe the work where there is a lack of certainty with respect to key input data. This uncertainty is modeled with either a set of scenarios or with interval data, and the resulting models are solved with a proposed ‘‘robustness approach’’.

Bertsimas and Thiele (2006) explore the current status of static robust optimization for LP problems. These approaches do find a solution over all time periods. When the planning is dynamic on a rolling horizon, it is reasonable to expect that better solutions can be found, as we can dynamically adjust the planning when more information is known. In the dynamic planning situation it is assumed that there are two sets of variables. One set must be determined before all the parameters are determined, and the other set of variables model future decisions that need not be determined until a later stage. Ben-Tal et al. (2004) introduce a computationally tractable robust formulation for the special case when the future decision variables can be expressed with an affine function of the uncertainty set. Although they observe many situations where this assumption holds, it is not an easy task to verify if the problem at hand satisfies all the requirements. The method has no flexibility in elaborations with uncertainty sets, since a minor adjustment could change the robust counterpart into an intractable formulation. Bertsimas and Caramanis (2008) approach a more general problem where the uncertainty set may be a general polytope. In the solution approach, they use a partitioning of the uncertainty set and find a static robust solution for each partition. In a later stage, without uncertainty, at least one of the static solutions fulfills the now realized parameters, and the best static solution is selected for implementation. The difficulty with this approach is to select a well-performing partitioning so that the static robust solutions are reasonable, while at the same time keeping the number of partitions low for the sake of efficiency. Beck and Ben-Tal (2009) study dual problems associated with the robust counterparts of uncertain convex programs. They show that while the primal robust problem corresponds to a decision maker operating under the worst possible data, the dual problem corresponds to a decision maker operating under the best possible data. Chen and Zhang (2009) describe an adjustable robust counterpart to modeling and solving multi-stage uncertain linear programs with fixed recourse. The paper by Aissi et al. (2009) is one of few papers that discusses robust optimization for integer programming. In the paper, the authors present robust optimization results for some well-known combinatorial optimization problems where the uncertainty is in the objective function coefficients. The coefficients are assumed to either be one sample from a finite set of possible coefficients, or bounded by intervals on the real axis. A robust formulation of a multi-period inventory management problem is presented in See and Sim (2010). In the paper, the demand is assumed to be uncertain under a factor-based demand model, where the demand in one time period is dependent on random factors that are realized sequentially at later time periods. This (affine) demand model is designed to more accurately capture phenomena such as trends and seasons in common forecasting models, such as ARMA (AutoRegressive Moving Average) based models. However, it does not generalize to arbitrary polytope restrictions for the demand.

Adida and Perakis (2006) proposed a deterministic robust optimization formulation for dealing with demand uncertainty in a dynamic pricing and inventory control problem for a make-to-stock manufacturing system. They assume that the demand is a linear function of the prize. Also, no back-order are allowed. Werners and Wülfing (2010) describes how a robust approach for internal transports at Deutsche Post World Net has been implemented. The demand is assumed to be uncertain over the year and they propose robustness by measuring and penalizing deviations from an ideal scenario solution and a integrated model solution. Wagner (2010) study profit maximization in inventory control problems where demands are unknown. Here, the author studies online problems and describe a worst case metric where the online solution is compared with an off-line solution.

The main contribution of our paper is to present an efficient and practical solution method for robust optimization for specific linear programming problems in which the uncertainties can be described with an arbitrary polytope. No assumptions are required about the underlying distribution to describe the uncertainty in this formulation. Instead, we assume that characteristics in uncertainty are formulated preferably with affine constraints. For example, the overall uncertainty in a rolling horizon problem, as well as parts of the uncertainty, could be restricted, and can, therefore, be expressed with a linear constraint. We note that this introduces a dependency of the uncertainty parameters between the time periods. To the best of our knowledge, the explicit dependency between time periods is a complication not addressed in any previous work. This dependency is an important part of the solution approach proposed in this article. Moreover, the size of the model when uncertainties are included often increases drastically. In our proposed solution approach, the model size is kept at the same size as the corresponding deterministic model (with no uncertainty).