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Exploring the oscillatory dynamics of a forbidden returns inventory system

Xun Wang [a,*], Stephen M. Disney [b], Jing Wang [a]

[a] School of Economics and Management, BeiHang University, Beijing 100191, China

[b] Logistic Systems Dynamics Group, Cardiff Business School, Cardiff University, Colum Drive, Cardiff CF10 3EU, UK

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ABSTRACT

We present an analytical investigation of the intrinsic oscillations in a nonlinear inventory system where excessive inventory cannot be returned to the supplier. Mathematically this is captured by a non-negative constraint on the replenishment order. By studying the eigenvalues of the characteristic matrices of the system, the criteria for different types of dynamic behaviour (including convergence, periodicity, quasi-periodicity, chaos, and divergence) are derived. The upper and lower bounds of the order and inventory oscillations are found via a time-domain analysis. Our results are verified by bifurcation diagrams. We find that the closer the replenishment rule feedback parameters are to the convergence area, the milder the intrinsic oscillation of the system.

1. Introduction and motivation

A supply chain's inventory control policy needs to attenuate fluctuations in demand, so as to maintain a smooth production rate in the face of both externally and internally generated disturbances. It should also maintain inventory levels around target safety stock levels. One of the most well-studied oscillation phenomena in supply chains is the so-called bullwhip effect. Since the pioneering work of Lee et al. (1997), much effort has been devoted to this problem. Many factors affecting the bullwhip effect have been investigated including: the impact of forecasting methods (Chen et al., 2000; Dejonckheere et al., 2003); statistical modelling of demand processes (Aviv, 2003; Gaalman, 2006); cooperative mechanisms such as information sharing (Lee et al., 2000; Dejonckheere et al., 2004); and Vendor Managed Inventory (Disney and Towill, 2003a). The integration of control theory and system dynamics approaches provides a powerful approach for quantifying and mitigating such effect (Disney and Towill, 2003b). However, in most of the previous theoretical studies on the bullwhip effect, linear inventory system models were adopted. In linear systems dynamical oscillations can only be generated by external events (such as demand). This has greatly limited the applicability of published results and has made it impossible to explain and describe oscillations caused by internal factors.

To maintain linearity of inventory system models, order rates are permitted to take negative values. This means that all participants in a supply chain are allowed to return excess product freely. Specifically, a negative order rate value leads to a decrease in the inventory level at the consuming echelon and an immediate increase in the inventory level at the supplying echelon. This assumption may be difficult to realize in reality but we do recognize that it exists in some supply chains. For example in the consumer electronics and book publishing supply chains it is accepted practice that retailers may return unwanted product to the manufacturer/publisher. In practice this may also mean that the excess inventory is not physically moved from one location to another but instead will be considered to be in the possession of the upstream supplier until being used as part of a future replenishment (Hosoda and Disney, 2009).

It has also been demonstrated that nonlinear effects play an important role in inventory systems, sometimes even a dominant role (Nagatani and Helbing, 2004). When linearity assumptions are removed complex dynamic behaviours are revealed. The behaviour may even become chaotic or hyper-chaotic. More importantly, oscillations generated internally by the system itself, rather than by the external environment, may arise. Mosekilde and Larsen (1988) adapted the beer game model (Sterman, 1989) to include both forbidden returns and lost sales constraints. To make the chaotic phenomenon more obvious, a long lead-time was used. Mosekilde and Larsen (1988) found that the operating cost of this constrained system could be 500 times higher than its linear counterpart. Thomsen et al. (1992) concluded that economic and business systems do not necessarily operate close to

* Corresponding author. Tel.: +86 10 8233 8930.
E-mail addresses: paul.wong.buaa@gmail.com (X. Wang), DisneySM@cardiff.ac.uk (S.M. Disney), pekwjing@public.bta.net.cn (J. Wang).

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their steady state. Hwarng and Xie (2008) investigated several system factors that affect chaotic behaviour and discovered a ‘chaos-amplification’ phenomenon between supply chain echelons. Wu and Zhang (2007), using a supply chain model with a constrained discount rate and exponential demand function, found that the attractors of the model in the phase space moves with the assumed initial states, rendering it impossible to provide guidelines for avoiding chaos by bifurcation analysis. Wang et al. (2005) used the Lyapunov exponent to identify chaotic demand in real supply chain data and proposed an algorithm to cope with it from a time series aspect.

The piecewise linear modelling approach has also been shown to be effective for certain nonlinear supply chain problems as the piecewise linear function is able to approximate any nonlinear function to any required level of accuracy. Liu (2005) and Rodrigues and Boukas (2006) analyzed the stability of supply chain inventory systems with piecewise linear techniques. Laugesen and Mosekilde (2006) and Mosekilde and Laugesen (2007) studied border-collision bifurcations in piecewise linear supply chain systems. However, mathematical properties of such systems, such as local and global stability conditions and bifurcations, are still “very hard to investigate” and “notoriously challenging” (Sun, 2010).

This paper is concerned with identifying the range of oscillations in a constrained supply chain that are generated by the system itself rather than by the external environment. For our analysis a unit step demand input will be adopted (unless otherwise stated) to emphasize that it is the system itself rather than the environment that is generating these dynamical effects. The pattern and amplitude of each kind of oscillation will be characterized. Section 2 models the constrained one echelon supply chain system piecewise-linearly. Section 3 investigates the type of oscillation pattern produced by the inventory system. Section 4 focuses on the upper and lower bounds of oscillation with respect to the order volume and inventory level. Concluding remarks are given in Section 5.

2. Modelling and assumptions of forbidden returns inventory system

Our inventory system model has several components that can be described in the time domain by difference equations. These equations will be listed below. First, we assume that the inventory system uses exponential smoothing as a forecasting method. The exponential smoothing forecasts are generated with

$$\hat{d}_t = \alpha_F d_t + (1-\alpha_F)\hat{d}_{t-1}, \tag{1}$$

where d_t is the demand at time t, $\hat{d}_t$ is its forecast at time t and α_F is the exponential smoothing constant. $0 \le \alpha_F < 2$ is required for stability of the forecasting system. Within the inventory system, the inventory levels obey the usual conservation law such that the new level of inventory equals the inventory level in the previous period plus the net in-flow into that state. That is

$$i_t = i_{t-1} + c_t - d_t, \tag{2}$$

where i_t is the inventory level, c_t is the completion rate (what arrives from the supplier or the production system) and o_t is the order rate at time. Factors such as the loss/damage/late delivery of goods in storage and transportation will be omitted. The work-in-progress, WIP, (or orders placed but not yet received) also obeys the conservation law,

$$w_t = w_{t-1} + o_{t-1} - c_t = \sum_{i=1}^{T_p} o_{t-i}, \tag{3}$$

where w_t is the work-in-process level and T_p is the physical lead-time. To capture the time delay between placing an order and receiving it into inventory there is a sequence of events delay for (information) order processing of one period and a physical lead-time, T_p, for production/transportation, also of one period duration. This means that

$$r_t = o_{t-1}, \tag{4}$$

and

$$c_t = r_{t-1} \tag{5}$$

r_t is an auxiliary variable used to capture the review period and ensure a proper sequence of events. This auxiliary variable is also essential in establishing the matrix form of the inventory system. We make this unit lead-time assumption for the simplicity in future analysis. Non-negative WIP is assured as the orders cannot be negative; this fact is most easily recognized from the RHS of (3). It is clear from existing research on linear inventory system analysis that increasing the lead-time will severely harm the dynamic performance and reduce the size of its stability region in the parametrical space, Towill and Disney (2008). We note that it is theoretically possible to extend this analysis to higher lead-time cases. However the complexity of the analysis increases with the lead-time and, as we will demonstrate, this system already exhibits a very rich set of dynamics behaviours, even with such a short, known and constant lead-time.

Using these four building blocks (the forecast, the inventory and WIP balance equations and the lead-time equations), we adopt the Automatic Pipeline, Variable Inventory and Order Based Production Control System (APVIOBPCS) ordering policy for placing replenishment orders. For a thorough review of this policy and the entire IOBPCS family we refer readers to Sarimveis et al. (2008). This policy has been frequently studied as it is of a very general nature. This policy determines the replenishment order quantity as “the demand forecast, plus a fraction of inventory discrepancy, plus a fraction of work-in-process discrepancy” (Disney and Towill, 2003a). The APVIOBPCS replenishment rule is a generalization of the industrially popular Order-Up-To policy, Dejonckheere et al. (2003), and has a long history in the literature. Inventory and work-in-process discrepancies are the expected levels, $\hat{i}_t$ and $\hat{w}_t$, minus the actual levels, i_t and w_t, respectively. In the APVIOBPCS model, expected inventory level is set as a multiple k of expected demand (k is a constant called “target inventory gain”). The expected work-in-process is a multiple T_p of expected demand, $\hat{w}_t = T_p \hat{d}_t$ (Sarimveis et al., 2008). For simplicity we set $k=1$ and $T_p=1$. Therefore, $\hat{i}_t = \hat{w}_t = \hat{d}_t$. We use α_S and α_{SL} to denote the fractions arbitrarily set by decision makers. The subscript S is for stock (inventory), SL for supply line (work-in-process). In other words, α_S and α_{SL} are proportional feedback controllers acting upon the of inventory and work-in-process information used to generate a replenishment order. The APVIOBPCS ordering policy can be expressed as

$$\begin{aligned} o_t &= [\hat{d}_t + \alpha_S(\hat{i}_t - i_t) + \alpha_{SL}(\hat{w}_t - w_t)]^+ \\ &= [\hat{d}_t + \alpha_S(\hat{d}_t - i_t) + \alpha_{SL}(\hat{d}_t - w_t)]^+ \\ &= [(1+\alpha_S+\alpha_{SL})\hat{d}_t - \alpha_S i_t - \alpha_{SL} w_t]^+ \end{aligned} \tag{6}$$

The ordering policy in (6) was found to mimic real-life decisions made by players of the Beer Game, Sterman (1989). Forbidden returns (non-negative orders) are enforced with the maximum operator, $[x]^+ = \max[0, x]$, in (6). Contrary to the linear assumption, we assume that when the desired order rate (calculated by the expression inside the square bracket in (6)) is negative, the supply chain participant can only stop ordering and wait for the excess inventory to be depleted before positive orders are resumed. Since there is no non-negative constraint on the inventory level, the following underlying assumptions are