Int. J. Production Economics 144 (2013) 68–75

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

The heterogeneous fleet vehicle routing problem with overloads and time windows

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ARTICLE INFO

Article history:
Received 30 September 2011
Accepted 18 January 2013
Available online 29 January 2013

Keywords:
Vehicle routing
Heuristics
Time windows

ABSTRACT

This paper addresses a new variant of the vehicle routing problem (VRP) with time windows, in which the vehicle fleet comprises units of different capacities and some overloads (i.e., loading vehicles above nominal capacity) are allowed. Although often encountered in practice, the problem, which we call heterogeneous fleet vehicle routing with overloads and time windows (HFVROTW), has not been previously tackled in the literature. We model it by integrating the constraint of the total trip load into the objective function, and solve it via a sequential insertion heuristic that employs a penalty function allowing capacity violations but limiting them to a variable predefined upper bound. Computational results on benchmark problems show the effectiveness of the proposed approach in reducing vehicle costs with minimal capacity violations, thus offering evidence of the significance of this VRP variant.

1. Introduction

Real-life vehicle routing emanates in a large variety of cases where goods or people have to be moved between locations in specific time intervals by different transportation means (Golden et al., 2008). The academic community has studied several versions of vehicle routing and scheduling problems, proposing an immense list of solution approaches, ranging from simple heuristics to complex meta-heuristics and exact methods—see Gendreau and Tarantilis (2010) for a thorough review of past and recent developments in the field. Most researchers though, have concentrated on simplified instances of the problem, i.e., instances that do not accommodate constraints or objectives often encountered in practice, which guide the actual solutions in real vehicle routing and scheduling problems—see Kritikos and Ioannou (2010) for such a discussion.

In this work, we study the heterogeneous fleet vehicle routing with overloads and time windows (HFVROTW), motivated by the fact that real-life scheduling of vehicles involves, to some extent and in addition to time restrictions, some small capacity violations in most distribution scenarios or public transportation in urban or rural areas. Indeed, bus overloads in peak hours, truck overloads enforced by large product demand, or overloads in power and telecommunication networks, are often encountered in real-life but when approaching routing from a research perspective, these facts are always ignored.

The HFVROTW can be described as follows: Consider a heterogeneous fleet of vehicles, i.e., comprising vehicles with different capacities, located at a central depot (distribution center or transportation hub). The vehicles are required to serve a set of customers, which are geographically dispersed in the area covered by the depot. Each customer has a known demand and a time window for service. Also, there is a service time associated with each customer, and the distance between each pair of customers is known, as is the distance between all customers and the depot. In the solution of the HFVROTW, vehicles are allowed to carry load over their capacity at a penalty incurred in the total solution cost. Thus, in contrast to the classical vehicle routing problem, the goal of the HFVROTW is to minimize a combined objective of the total distance traveled by vehicles, the fixed costs of vehicles performing service, and the capacity violations of all vehicles included in the final schedule. An implicit decision embedded in the problem is the selection of the fleet's composition, i.e., how many vehicles of each available type (capacity) are selected for service.

To our knowledge, research related to the HFVROTW is non-existent. Most approaches deal either with: (a) the heterogeneous vehicle routing problem (HVRP—i.e., the problem with vehicles of different capacities) or (b) the heterogeneous vehicle routing problem with time windows (HVRPTW—i.e., the previous problem with time windows). Heuristic methods proposed for the HVRP include, as described in Golden et al. (1984), adaptations of the Clarke and Wright savings algorithms, the giant tour partitioning approach, the

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http://dx.doi.org/10.1016/j.ijpe.2013.01.020

matching based savings heuristics, the generalized assignment based heuristic, the sophisticated improvement based heuristic, composite heuristics, and a multi level composite heuristic for the multi-depot HVRP.

Liu and Shen (1999) were the first to tackle the HVRPTW and developed a number of parallel insertions heuristics based on the insertion scheme of Solomon, and embedding in the calculations of the relevant criteria the acquisition costs of Golden et al. (1984). Dullaert et al. (2002) proposed a sequential construction algorithm, extending Solomon's I1 heuristic with vehicle insertion savings calculations based again on the criteria of Golden et al. (1984). Dondo and Cerda (2007) proposed a 3-phase algorithm for the multi-depot HVRPTW motivated by cluster-based optimization, while Paraskevopoulos et al. (2008) presented a two-phase solution framework relying on a hybridized tabu search integrated within a new reactive variable neighborhood search meta-heuristic algorithm, with very good results. Braysy et al. (2008) presented a deterministic annealing metaheuristic for the HVRPTW, outperforming the results of Liu and Shen (1999), and Braysy et al. (2009) developed a linearly scalable hybrid threshold-accepting and guided local search meta-heuristic for solving large scale HVRPTW instances. Finally, Repoussis and Tarantilis (2010) proposed an Adaptive Memory Programming solution approach for the HVRPTW that provides very good results in the majority of the benchmark instances examined.

In a relevant research thread, Rochat and Semet (1994) developed a tabu search approach for the HVRPTW, which takes into account the drivers' breaks and possible accessibility restrictions. Brandao and Mercer (1997) developed also a tabu search for the multi-trip vehicle routing and scheduling problem, in which each vehicle can make several trips per day, while access can be restricted for some vehicles to some customers; the algorithm the authors proposed allows for both weight and volume capacity restrictions on the vehicles.

In this work, we address for the first time the vehicle routing problem with time windows when the fleet is heterogeneous, i.e., comprises vehicles of different capacities and associated costs, and overloads are allowed up to a pre-specified bound, at a penalty though embedded in the problem's objective function. The penalty, which is a measure of the deviation of the actual load from the nominal vehicle capacity, is similar to the one presented by Gheysens et al. (1984), while the capacity bound varies in order to examine a large area of the potential solution space. For the solution of the HFVROTW we propose a simple solution method, i.e., a sequential insertion heuristic, extending the traditional insertion criteria of Solomon (1987), and adapting Golden et al.'s (1984), Dullaert et al.'s (2002), and Liu and Shen's (1999) ones. The computational results on benchmark problems reinforce our intuition for practical applicability of the proposed approach, with minimal adverse effects on vehicle loads and positive impact on total costs.

The remainder of the paper is organized as follows. In Section 2 the HFVROTW is formulated. Section 3 offers the basic contribution of our work, i.e., the way we devise and employ the overload penalty function, the criteria we use within the solution schemes, and the overall solution approach we propose. Section 4 presents an illustrative example based on sample literature instances, and Section 5 includes computational results on benchmark problems. Finally, Section 6 provides our concluding remarks and suggestions for future research.

2. Mathematical model

The HFVROTW can be stated as follows: Find a set of closed routes, for a fleet of T vehicles with known capacities $C_1, C_2, \ldots, C_T$, servicing a set of $|L|-1=n-1$ customers, from a central depot at minimum cost. L is the set of customers including the depot, which is a distinct node of the underlying connected graph. Indices i, j and u refer to customers and take values between 2 and n, while index $i=1$ refers to the depot; an additional index k counts the vehicles. Vehicles are initially located at the central depot. Each customer i poses a demand q_i, requires a service time, s_i, has a time window $[e_i, l_i]$, and is serviced by exactly one vehicle. There is a cost c_{ij}^k, (related to the travel time t_{ij}^k and distance d_{ij}) associated with the path from customer i to customer j, using vehicle k. Furthermore, a fixed acquisition cost f_k is incurred for each of vehicle k in the routes. Each route originates and terminates at the central depot and must satisfy the time window constraints, i.e., a vehicle cannot start servicing customer i before e_i and after l_i; however, the vehicle can arrive before e_i and wait for service. Note that capacity constraints are relaxed in the HFVROTW.

Gheysens et al. (1984) presented a mathematical programming formulation for the HVRP. We extend their model to formulate the HFVROTW, the variables of which include: (a) the arrival–departure time to/from customer i, respectively, denoted by a_i and p_i for each customer I; (b) the vehicle load Q_k; (c) he sequence in which vehicles visit customers, x_{ij}^k, and (d) the activation of a vehicle k, z_k. Variables (c) and (d) are defined as follows:

$$x_{ij}^k = \begin{cases} 1, \text{ if vehicle } k \text{ travels from } i \text{ to } j \\ \text{and} \\ 0, \text{ otherwise} \end{cases} \tag{1}$$

$$z_k = \begin{cases} 1, \text{ if vehicle } k \text{ is active} \\ 0, \text{ otherwise} \end{cases} \tag{2}$$

Given the above-defined variables the HFVRPOTW can be formulated as follows:

$$\text{Minimize} \quad \sum_{k=1}^{T}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}^k + \sum_{k=1}^{T} f_k z_k + \sum_{k \in T*} \lambda_k (Q_k - C_k) \tag{3}$$

Subject to:

$$\sum_{i=1}^{n}\sum_{k=1}^{T} x_{ij}^k = 1, \quad \forall j = 2,3,\ldots,n \tag{4}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{T} x_{ij}^k = 1, \quad \forall i = 2,3,\ldots,n \tag{5}$$

$$x_{ij}^k \le z_k, \quad \forall i,j = 1,2,\ldots,n, \quad \forall k = 1,2,\ldots,T \tag{6}$$

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$$\sum_{j=2}^{n} x_{1j}^k \le 1 \quad \forall k = 1,2,\ldots,T \tag{7}$$

$$\sum_{i=2}^{n} x_{i1}^k \le 1 \quad \forall k = 1,2,\ldots,T \tag{8}$$

$$\sum_{i=1}^{n} x_{iu}^k - \sum_{j=1}^{n} x_{uj}^k = 0, \quad \forall k = 1,\ldots,T, \quad \forall u = 2,\ldots,n \tag{9}$$

$$a_j \ge (p_i + t_{ij}) - (1 - x_{ij}^k)M, \quad \forall i,j = 1,2,\ldots,n, \quad \forall k = 1,2,\ldots,T \tag{10}$$

$$a_j \le (p_i + t_{ij}) - (1 - x_{ij}^k)M, \quad \forall i,j = 1,2,\ldots,n, \quad \forall k = 1,2,\ldots,T \tag{11}$$

$$a_i \le p_i - s_i, \quad \forall i = 1,\ldots,n \tag{12}$$