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Optimal control of a production-inventory system with product returns

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ABSTRACT

We consider a production-inventory system that consists of n stages. Each stage has a finite production capacity modelled by an exponential server. The downstream stage faces a Poisson demand. Each stage receives returns of products according to independent Poisson processes that can be used to serve demand. The problem is to control production to minimize discounted (or average) holding and backordering costs. For the single-stage problem ($n=1$), we fully characterize the optimal policy. We show that the optimal policy is base-stock and we derive an explicit formula for the optimal base-stock level. For the general n-stage problem, we show that the optimal policy is characterized by state-dependent base-stock levels. In a numerical study, we investigate three heuristic policies: the base-stock policy, the Kanban policy and the fixed buffer policy. The fixed-buffer policy obtains poor results while the relative performances of base-stock and Kanban policies depend on bottlenecks. We also show that returns have a non-monotonic effect on average costs and strongly affect the performances of heuristics. Finally, we observe that having returns at the upstream stage is preferable in some situations.

1. Introduction

The importance of product returns is growing in supply chains. Customers often can return products a short time after purchase, due to take-back commitments of the supplier. For instance, the proportion of returns is particularly important in electronic business where customers cannot touch a product before purchasing it. Customers might also return used products a long time after purchase. This type of return has increased in recent years due to new regulations on waste reduction, especially in Europe. Some industries also encourage returns for economical and marketing reasons. Though different in nature, these two types of returns are similar from an inventory control point of view since they constitute a reverse flow which complicates decision making.

The inventory control literature on product returns is quite abundant (see e.g. Fleischmann et al., 1997; Ilgin and Gupta, 2010; Zhou and Yu, 2011). However, most of the literature focusses on single-echelon systems with infinite production capacity. In this paper, we fill this gap by considering a n-stage production/inventory system with finite production capacity and product returns at each stage (see Fig. 1). The flow of returns at the finished good (FG) inventory may result from remanufacturing, recycling, repairing or simply returning new products. The flows of returns at the work-in-process (WIP) inventories can also result from disassembly operations. For instance, the Kodak company reuses only some parts of cameras like circuit board, plastic body and lens aperture (Toktay et al., 2000).

More precisely, we adopt a queueing framework to model production capacity. Items are produced by servers one by one and each unit requires a random lead-time to be produced. We assume that each stage consists of a single exponential server and an output inventory. The downstream stage faces a Poisson demand. Each stage receives returns of products, according to independent Poisson processes, that can be used to serve demand. The problem is then to control production at each stage, in order to minimize discounted/average holding and backordering costs. We also study the single-stage problem which has not been studied in the literature. In what follows, we review the literature on single-echelon and multi-echelon systems with returns, before presenting in detail our contributions.

The literature on single-echelon systems is quite mature. Heyman (1977) considers an inventory system with independent Poisson demand and Poisson returns. Unsatisfied demands are backordered. Heyman assumes zero lead-times and linear costs for both manufacturing and remanufacturing. These strong assumptions imply that the optimal production policy is a make-to-order policy and that the optimal disposal policy is a simple threshold policy: when the inventory level exceeds a certain disposal threshold R, every returned item is disposed upon arrival. An explicit expression for the optimal disposal threshold is also derived. For a lost sale problem with exponential

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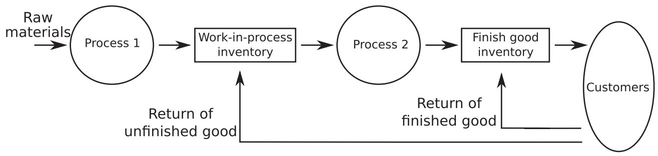

Fig. 1. A two-stage production/inventory system with returns.

service times, Poisson demand and returns, Zerhouni et al. (in press) investigate the impact of ignoring dependency between demands and returns.

Fleischmann et al. (2002) consider a similar setting with deterministic manufacturing lead-time and fixed order cost. Again, remanufacturing lead-time and remanufacturing costs are neglected. They extend results standing for a system without returns by showing that the optimal policy is (s,Q) for the average-cost problem. For the periodic review version with a stochastic demand either positive or negative in each period, Fleischmann and Kuik (2003) show the average-cost optimality of an (s,S) policy. Simpson (1978) and Inderfurth (1997) consider a periodic-review problem where returns are held in a separate buffer until they are remanufactured or disposed of. When the remanufacturing lead-time is equal to the production lead-time and the costs are linear, they show that a three-parameter policy is optimal.

Apart from these optimal control papers, several heuristic policies have been investigated in the literature. Van der Laan et al. (1996b) model the remanufacturing shop as an $M/M/c/(c+N)$ queue with c parallel servers and introduce the (s_p,Q_p,N) policy where s_p is the reorder point, Q_p the order quantity and any return is disposed whenever the number of products waiting for repair equals N. Van der Laan et al. (1996a) extend this policy with the (s_p,Q_p,s_d,N) policy where returns are disposed when the stock level is above s_d. Van der Laan and Salomon (1997) consider a model with correlated demand process and return process. They compare an (s_p,Q_p,Q_r,s_d) push-disposal policy with an (s_p,Q_p,s_r,S_r,s_d) pull-disposal policy to coordinate manufacturing and remanufacturing decisions. For the push-disposal policy, returned products are remanufactured with batch size Q_r. For the pull-disposal policy, remanufacturing is initiated only when the finished good inventory is below s_r and the remanufacturable inventory is above S_r. Teunter and Vlachos (2002) complement the numerical study of the above model.

The literature on multi-echelon systems with returns is much more limited. In their seminal work (without returns), Clark and Scarf (1960) studies a series inventory system with n stages, periodic review, linear holding and backorder cost, no setup cost and stochastic demand at the downstream stage. They prove that a base-stock policy is optimal. DeCroix et al. (2005) extend the results of Clark and Scarf (1960) to the case where demand can be negative. They also propose a method to compute a near optimal policy, explain how to extend their model when returns occur at different stages and compare the base-stock policies to fixed-buffer policies. DeCroix (2006) combines the multi-echelon structure of DeCroix et al. (2005) and the remanufacturing structure of Inderfurth (1997). DeCroix and Zipkin (2005) and DeCroix et al. (2009) consider assemble-to-order systems with returns of components or finished product.

In production-inventory systems, replenishment is modelled in a different way than in pure inventory systems. Items are produced by servers one by one, or possibly by batches. Each unit, or batch, requires a random lead-time to be produced. Hence replenishments are capacitated in production-inventory systems. In line with this approach, Veatch and Wein (1992) consider a n-stage system with exponential server at each stage. Otherwise, their assumptions are similar to Clark and Scarf (1960). They prove that the optimal policy is never a state-dependent base-stock policy. In another paper, Veatch and Wein (1994) studies the case $n=2$. They investigate several classes of policies and compare them to the optimal policy. They conclude that the base-stock policy is generally the best heuristic. However, when the downstream station is the bottleneck, the Kanban policy is better. Dallery and Liberopoulos (2003) investigates a generalized Kanban policy being a mix between Kanban and base-stock policy. In a deterministic environment, several papers have investigated capacitated production and/or remanufacturing (see e.g. Nahmias and Rivera, 1979; Teunter, 2001, 2004; Li et al., 2007).

In this paper, we extend the model of Veatch and Wein (1992) by including Poisson returns at each stage. We show that the optimal policy is still a complex state-dependent base-stock policy and we derive several monotonicity results for the base-stock levels. Interestingly, the single-echelon problem has not been treated in the literature, when including Poisson returns. In this case, the optimal policy reduces to a simple base-stock policy and we are able to derive an explicit formula for the optimal base-stock level for both average-cost and discounted-cost problems. Such explicit formulas are very rare in inventory control theory, especially when returns are included. When service times, inter-arrival times and inter-return times are not exponential but have general i.i.d. distributions, we explain how to compute the optimal base-stock level by using results from the newsvendor problem.

The optimal policy of the n-stage problem has a complex form and might be difficult to implement in practice. To counter this, we evaluate the performances of three classes of heuristic policies (fixed buffer, base-stock and Kanban) which are reasonable with respect to the optimal policy structure. The fixed-buffer policy obtains poor results while the relative performances of base-stock and Kanban policies depend on bottlenecks, consistently with Veatch and Wein (1996). Moreover, we observe that return rates strongly affect the relative performances of heuristics.

Section 2 describes in detail the n-stage problem. Section 3 provides a full characterization of the optimal policy for the single-stage system. Section 4 shows that the optimal policy for the n-stage system is a state-dependent base-stock policy. Section 5 investigates the performances of three heuristic policies. Finally, we conclude and discuss avenues for research in Section 6.

2. Assumptions and notations

We consider a n-stage production/inventory system in series which satisfies end-customer demand (see Fig. 2). Station M_i, $i \in \{1, \dots, n\}$, produces items one by one. The production lead-time of station M_i is exponentially distributed with rate μ_i. Preemption is allowed and works as follows. The processing of a job at station M_i can be interrupted at any point in time and continued latter. Because of the memoryless property of the exponential distribution, continuing a job is equivalent to restarting it from the beginning. Produced items are stocked in a buffer B_i just after M_i. The end buffer B_n sees customer demands arriving according to a Poisson process with rate λ. We assume that backorders are allowed. At time t, the on-hand inventory at B_i $(1 \le i < n)$ is denoted by $X_i(t)$ and the net on-hand inventory at B_n is denoted

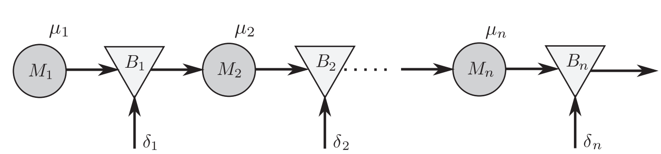

Fig. 2. The n-stage $M/M/1$ make-to-stock queue with product returns.