Inventory control in a lost-sales setting with information about supply lead times

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ABSTRACT

Supply chain collaboration using advancements in information technology is on the rise and this includes sharing of information between suppliers and buyers. In this paper we study the value of information about the development of supply lead times from a buyer's perspective. We consider a periodically reviewed single-item inventory system in a lost sales setting where at most one order can be outstanding at a time. We compare the performance of an inventory model assuming informed lead times to a model assuming uninformed independent and identically distributed lead times. We employ the dynamic programming approach to find the best state-dependent ordering policy to minimize the expected average total cost per time unit. Our numerical results show that acquiring information about the development of supply lead times is of value. In general the best policy suggested by the model assuming informed lead times causes lower average cost than the model assuming uninformed lead times.

1. Introduction

The crucial part of inventory management is to make replenishment decisions in the face of uncertainties at upstream and downstream stages of the supply chain. Demand uncertainty represents uncertainty in the downstream supply chain, whereas lead-time uncertainty captures the uncertainty of the supply system, i.e. the upstream part of the supply chain. As discussed in an empirical examination by Wagner and Bode (2008), such variations can be equated to supply chain risks which lower its performance. One way to mitigate supply chain risk is by improving confidence between decision-makers through collaboration and better information sharing (Christopher and Lee, 2004). An empirical study by Li et al. (2009) suggests that effective use of information technology has a positive effect on supply chain integration and hence on its performance. Advancements in information technology and its role in supply chain management provides managers with an opportunity to relatively easily obtain dynamic information about demand and supply variations. Similar to demand variations, major variations in supply lead times may have identifiable sources, such as equipment breakdown and workload conditions. These sources of variation reflect the condition of the supply system, and replenishment lead times evolve as the system evolves over time. Inventory models which consider the evolution of demand and use advance information in decision making are widely studied, for example Hariharan and Zipkin (1995), Karlin and Fabens (1959), Song and Zipkin (1993) and Gallego and Özer (2001). However, more sparsely studied are the models with shared information about upstream supply conditions. The hypothesis test by Cannon and Homburg (2001) shows that an increasing number of suppliers share information to reduce acquisition and operation costs for the customer, thus emphasizing a need for further study of such systems.

Since the mid-1990s some results for inventory control models with information about supply conditions have been available, such as Song and Zipkin (1996). In these models, stockouts are backordered. However, stockouts may also result in lost sales. The lost-sales case has considerable practical significance. The study by Corsten and Gruen (2004) shows that in the retail industry unmet demands result in lost sales not backorders in almost half of the cases. Lost sales also appear to be a common mechanism for handling shortages in some spare parts industries.

The purpose of this paper is to investigate the performance of the value of advanced lead-time information under lost sales. We study a single-item inventory control model with lost sales and ubiquitous demand uncertainty, controlled by a periodic review inventory replenishment policy with at most one order outstanding at any given time. The inventory is supplied by a system with evolving replenishment lead times which are informed in advance and with certainty at every ordering decision epoch.

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The assumption of no more than one outstanding replenishment order at a time can be justified from a practical standpoint. The likelihood of placing a new order while awaiting the arrival of an existing order can be negligible due to, for example, terms in the supplier-buyer contract or due to the buyer's internal ordering policy. Moreover, allowing for simultaneously outstanding orders makes the model numerically intractable in general (Zipkin, 2008a, 2008b).

We assume that the supply system is exogenous, i.e. its development is independent of our demands and replenishment orders (Zipkin, 2000, Section 7.4). Hence, replenishment order lead time is independent of the size of our order and only depends on the conditions of the supply system. However, the system is transparent in the sense that it is assumed to be possible to obtain information about its current state and about the development of its supply lead times. Hence, changes in lead times are characterized by a Markov chain. Thus, with a reliable estimate of current lead time, information of the development of future lead times is available through the Markovian dependence. The performance of the model with information about the development of supply lead times is compared to the model with independent and identically distributed (i.i.d.) lead times. The latter model represents the case where the supplier is not sharing information about the development of the supply lead times.

We consider an infinite time horizon inventory problem and formulate it as a dynamic program with finite and discrete state space. We assume that ordering, inventory holding and lost sales penalty costs are linear and the objective is to minimize expected average total cost per time unit over an infinite time horizon. Through numerical experiments, we observe the significance of information about the development of lead times. By changing the evolutionary pattern, we are able to study its qualitative effect on the inventory performance assuming either informed lead times or i.i.d. lead times. Such comparisons enable us to estimate the value of information available about the development of lead times compared to knowing only their long-term distribution.

The rest of the paper is organized as follows. In Section 2 we give a brief survey of the literature related to inventory systems with lost sales and information about supply conditions. Analytical models used for our numerical study are introduced in Section 3 and computational experiments are specified and reported in Section 4. Finally, Section 5 summarizes the results and concludes the paper.

2. Literature review

Davis (1993) contains a general discussion about the role of uncertainty in supply chain management from a business perspective by using Hewlett-Packard as a case study. Not only upstream and downstream uncertainties complicate the management of inventory but the way to treat stockouts also further complicates the problem. Inventory models with backlogging of unfulfilled demand are reasonably well understood today and detailed discussion about them can be found in Zipkin (2000) and Axsäter (2006). However, inventory control models in which stockouts are treated as lost sales have been studied more sparsely. For further insight into the differences between models with backorders and lost sales, we can refer to Montgomery et al. (1973) who study a model in which, during a stock out, a fraction of the demand is backordered and the rest is lost. The basic problem of an inventory model with lost sales (constant lead time) was formulated more than 50 years ago by Karlin and Fabens (1959). Yet, today it proves to be a difficult problem to work with because of the rapidly growing state space with longer lead times, which is discussed in Zipkin (2000, Section 9.6.5) and

described by the *curse of dimensionality*. Hence, few extensions to more complex systems are available.

Hadley and Whitin (1963, Section 4.11) discuss the difficulty in analyzing a continuously reviewed reorder point policy under lost sales. For results regarding periodically reviewed reorder point policies under lost sales see Johansen and Hill (2000). Nahmias (1979) provides approximations for the periodically reviewed inventory model with non-linear ordering cost and variable lead times, while Cohen et al. (1988) present an approximation for the periodically reviewed (s,S) inventory model with two priority demand classes.

Downs et al. (2001) study a multi-item periodically reviewed model with deterministic delivery time lags and a finite horizon. Recently, Zipkin (2008b) has presented an elegant state-space reduction technique for inventory models with lost sales and extended this approach to important variations of the model: limited capacity, correlated demands, stochastic lead times, and multiple demand classes. Equipped with this state-space reduction technique, Zipkin (2008a) is able to obtain optimal replenishment policies for longer lead times than reported before. His paper also tests plausible heuristics for a limited range of systems.

Due to the relatively easy-to-handle structure of inventory models with backordering they have usually been considered in past studies assuming more complex settings, such as information about supply conditions. This is the case, for example, in Song and Zipkin (1996), where exogenous supply is modeled as a Markov chain and as the information about supply evolves so do lead times. They analyze the effect of evolving lead times on policy decisions. Özekici and Parlar (1999) assume that the order is satisfied immediately if the supplier is available and, in the other extreme, if the supplier is unavailable, the order is never fulfilled. Availability of the supplier depends on the environment, which is modeled as a Markov chain. Gallego and Hu (2004) analyze inventory problems with Markov-modulated supply with limited capacity as well as separately Markov-modulated demand processes. Arifoğlu and Özekici (2010) extend this model and its results by considering the case where available information is imperfect due to a partially observable environment.

The paper by Ben-Daya and Hariga (2004) discusses the single-vendor–single-buyer production inventory model which minimizes the consolidated expected total cost per time unit for vendor and buyer. They assume that the lead time of an order depends on its size. A comment on this paper by Glock (2009) takes the model a step further by demonstrating benefits in considering different reorder points for each batch shipment and unequal-sized batch shipments. A note on the same paper by Hsiao (2008) proposes a variation of the original model by assuming two different reorder points and service levels.

For the reasons mentioned above, inventory models under lost-sales settings and with information about supply conditions have received limited attention. Arreola-Risa and DeCroix (1998) explore the model with supply durations occurring randomly and lasting for a random duration. Their model assumes that the demand is stochastic and delivery lead times are equal to zero. In case of stockouts, a fraction of the demand is backordered and the remaining fraction is lost. In the paper by Mohebbi (2003), he investigates the issue of random supply interruptions (available/unavailable) in a continuous review inventory system where demand as well as non-zero lead times are stochastic and where stockouts are lost sales. He assumes that the duration of supplier availability and unavailability is independent. It is also assumed that the maximum number of outstanding replenishment orders is limited to one at any time. Variations of this model are presented by Mohebbi (2004) and (Mohebbi and Hao, 2006). In all these models, the assumption is that supply interruptions and the duration are random and independent. Hence, although information about availability of the supplier is