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On monitoring process variability under double sampling scheme

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ABSTRACT

The presence of variation in all manufacturing and measurement processes is a natural phenomenon and is the key factor which affects the performance of all types of processes. A better understanding of the causes of variability in any processes is necessary to improve the process. For an efficient monitoring of process variability, we have suggested a set of variance type control charts based on auxiliary characteristics and evaluated their performances in terms of Average Time to Signal (*ATS*) (the performance measure at every point of variability shift) and Average Extra Quadratic Loss (*AEQL*) (the performance measure over the whole process shift range) under normal and gamma process environments. We have also examined the effects of contaminated environments on the *ATS* performance of different variance based charting structures. Illustrative examples on some selective variance type control structures are also provided for procedural details. Finally we have closed with concluding remarks about this study.

1. Introduction

Control Chats are important tool of Statistical Process Control (SPC) tool kit that are used to monitor different processes. The location/dispersion charts are used to monitor the process mean/variation respectively. Due to the fact that the presence of variability is sure in all manufacturing and measurement processes, the analysis of process variability (instability of a process) is an important aspect to process monitoring. An increased number of defective items in a process may be due to high variation while the less number of defective items may indicate that there is less variability in process and the items are closer to required standard (cf. Acosta-Mejia et al., 1999). The process variation can be classified as natural and un-natural. A process is said to be out-of-control if there is wide variability and inconsistent results due to presence of some un-natural variation in the process otherwise the process is said to be in-control.

Control charts help to study the in-control and out-of-control processes situations for different parameters in process i.e. location, dispersion etc and Shewhart, CUSUM and EWMA are three major types of control charts used to detect different type of shifts in process. A few of these types of control charts to monitor process variability can be referred from literature as: Garcia-Diaz (2007) proposed an effective variance control chart in case of missing data. Ghute and Shirke (2008) used a combination of *S*-chart and the conforming run length chart to develop a variance chart. Chen and Huang (2005) proposed a synthetic control chart for monitoring the process standard deviation of a normally distributed process. Some Shewhart type control charts for monitoring processes variability can be referred to Duncan (1986), Montgomery (2009), Khoo (2004), Chang and Gan (2004), Riaz (2008) and Tzong-Ru (2009). The auxiliary information based control charts have been used in different patterns to monitor a quality characteristic in improved manners (cf. Hawkins, 1993; Shu et al., 2005; Riaz, 2008; Riaz and Does, 2009).

Khoo and Quah (2004), Reynolds and Kim (2007), Abbasi et al. (2009) and Costa and Machado (2009) proposed multivariate control chart designs for monitoring the process mean and variability. Surtihadi et al. (2004) suggested new techniques for formulating both multivariate Shewhart and CUSUM charts for process dispersion. More research work dealing with CUSUM charts can be found in literature for example Hawkins (1981), Howell (1987), Chang and Gan (1995), Jiujun et al. (in press), Ou et al. (2012b) and Yang et al. (2012). Sheu and Tai (2006) proposed Generally Weighted Moving Average (GWMA) control chart which is more sensitive than the EWMA control chart in monitoring the process variability. Some of the related work on EWMA type charts for monitoring the process variability can be referred to Crowder and Hamilton (1992), Macgregor and Harris (1993), Yeh et al. (2003), Costa and Rahim (2004), Penghui and Zhang (2004), Vargas and Lagos (2007), Eyvazian

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et al. (2008), Machado and Costa (2008), Zhang et al. (2009, 2010), and Ou et al. (2011).

The existing literature on auxiliary based variability control charters namely V_r and V_t charts (cf. Riaz, 2008; Riaz and Does, 2009) which are investigated for normally distributed processes (without considering any contaminated environment) using simple random sampling technique. This study suggests variability control charts based on different variants of single auxiliary variable and extended version of two auxiliary variables under simple random sampling and double sampling schemes for normal as well as gamma distributed process environments. The cases of known as well as the estimated parameters of auxiliary characteristics under uncontaminated and contaminated process environments are also considered in this study. The performance of potential control charts is evaluated in terms of Average Time to Signal (*ATS*) (the performance measure at specific variability shift) and Average Extra Quadratic Loss (*AEQL*) (the performance measure over the whole process shift range) under two situations: (i) normal and gamma distributed uncontaminated process, and (ii) contaminated process environments. For this purpose we have considered the quality characteristics following trivariate normal distribution and trivariate gamma distribution.

The flow of the rest of article is given as: Section 2 describes double sampling scheme and the control charting structures of variance type Shewhart charts considered in this study; Section 3 contains the performance evaluations and comparisons among different variance type control charts under uncontaminated and contaminated environments; Section 4 provides illustrative examples and finally we conclude the findings of our study in Section 5.

2. Preliminaries and control charting structures

Let Y be the study variable and X, Z be two auxiliary variables of a population U with size N (which is generally very large or infinite even) and σ_y^2, σ_x^2 and σ_z^2 be the population variances of Y, X and Z, respectively. Let y_i', x_i' and $z_i'(i=1,2,\ldots,n)$ denote the values of the units included in phase-I sample $s_n(s_n \subset U)$ of size n is drawn by simple random sampling to observe X and Z in order to get the estimates of σ_x^2 and σ_z^2 as $s_x'^2$ and $s_z'^2$, respectively. Given s_n; y_i, x_i and $z_i(i=1,2,\ldots,m)$ denote the values of the units included in phase-II sample $s_m(s_m \subset s_n)$ of size m is drawn to observe Y, X and Z to get s_y^2, s_x^2 and s_z^2.

Let $e_y = \frac{s_y^2-\sigma_y^2}{\sigma_y^2}, e_x = \frac{s_x^2-\sigma_x^2}{\sigma_x^2}, e_z = \frac{s_z^2-\sigma_z^2}{\sigma_z^2}, e_x' = \frac{s_x'^2-\sigma_x^2}{\sigma_x^2}$ and $e_z' = \frac{s_z'^2-\sigma_z^2}{\sigma_z^2}$ then we have, $E(e_i)=0$,

$$E\left(e_y^2\right) = \lambda_m(U_{400}-1), E\left(e_x^2\right) = \lambda_m(U_{040}-1),$$
$$E\left(e_z^2\right) = \lambda_m(U_{004}-1), E\left(e_x'^2\right) = \lambda_n(U_{040}-1),$$
$$E\left(e_z'^2\right) = \lambda_n(U_{004}-1),$$

Table 1
Control charting structures.

Charting statistic	Mean (g_i) and Standard Deviations (d_i)	Control limits
$V_1 = s_y^2$	$g_1 = \sigma_y^2$ and $d_1 = \sigma_y^2\sqrt{\lambda_m(\mu_{400}-1)}$	
$V_2 = \frac{s_y^2}{s_x^2}\sigma_x^2$	$g_2 = \sigma_y^2[\lambda_m(\mu_{400}-\mu_{220})+1]$ and $d_2 = \sigma_y^2\sqrt{\lambda_m[(\mu_{400}-1)+(\mu_{040}-1)-2(\mu_{220}-1)]}$	
$V_3 = \frac{s_y^2}{\sigma_x^2}s_x^2$	$g_3 = \sigma_y^2[\lambda_m(\mu_{220}-1)+1]$ and $d_3 = \sigma_y^2\sqrt{\lambda_m[(\mu_{400}-1)+(\mu_{040}-1)+2(\mu_{220}-1)]}$	
$V_4 = s_y^2\exp\left(\frac{\sigma_x^2-s_x^2}{\sigma_x^2+s_x^2}\right)$	$g_4 = \sigma_y^2[0.5\lambda_m(0.75\mu_{040}-\mu_{220}-0.25)+1]$ and $d_4 = \sigma_y^2\sqrt{\lambda_m[(\mu_{400}-1)+0.25(\mu_{040}-1)-(\mu_{220}-1)]}$	
$V_5 = s_y^2\exp\left(\frac{s_x^2-\sigma_x^2}{s_x^2+\sigma_x^2}\right)$	$g_5 = \sigma_y^2[0.5\lambda_m(\mu_{220}-0.25\mu_{040}-0.75)+1]$ and $d_5 = \sigma_y^2\sqrt{\lambda_m[(\mu_{400}-1)+0.25(\mu_{040}-1)+(\mu_{220}-1)]}$	
$V_6 = s_y^2\left[W\exp\left(\frac{\sigma_x^2-s_x^2}{\sigma_x^2+s_x^2}\right)+(1-W)\exp\left(\frac{s_z^2-\sigma_z^2}{s_z^2+\sigma_z^2}\right)\right]$	$g_6 = \sigma_y^2[0.5\lambda_m W(0.75\mu_{040}-\mu_{220}-0.25) -0.5\lambda_m(1-W)(0.25\mu_{004}-\mu_{202}+0.75)+1]$ and $d_6 = \sigma_y^2\sqrt{\lambda_m\left[\begin{matrix}(\mu_{400}-1)+0.25W^2(\mu_{040}-1)+0.25(1-W)^2\\(\mu_{004}-1)-W(\mu_{220}-1)+(1-W)(\mu_{202}-1)\\-0.5W(1-W)(\mu_{022}-1)\end{matrix}\right]}$ where $W = \frac{\mu_{004}+2(\mu_{220}+\mu_{202})+\mu_{022}-6}{\mu_{040}+\mu_{004}+2\mu_{022}-4}$	K-Sigma limits $LCL_k = g_i-Kd_i$ $CL_k = g_i$ $UCL_k = g_i+Kd_i$
$V_7 = \frac{s_y^2}{s_x^2}s_x'^2$	$g_7 = \sigma_y^2[(\lambda_m-\lambda_n)(\mu_{400}-\mu_{220})+1]$ and $d_7 = \sigma_y^2\sqrt{[\lambda_m(\mu_{400}-1)+(\lambda_m-\lambda_n)\{(\mu_{040}-1)-2(\mu_{220}-1)\}]}$	Probability limits $LCL_p = V_{il}$ with $F(V_i = V_{il}) \le \alpha_l$ $CL_p = V_{ic}$ $UCL_p = V_{iu}$ with $F(V_i = V_{iu}) \ge 1-\alpha_u$
$V_8 = \frac{s_y^2}{s_x'^2}s_x^2$	$g_8 = \sigma_y^2[(\lambda_m-\lambda_n)(\mu_{220}-1)+1]$ and $d_8 = \sigma_y^2\sqrt{[\lambda_m(\mu_{400}-1)+(\lambda_m-\lambda_n)\{(\mu_{040}-1)+2(\mu_{220}-1)\}]}$	where $i = 1,2,\ldots,11$
$V_9 = s_y^2\exp\left(\frac{s_x'^2-s_x^2}{s_x'^2+s_x^2}\right)$	$g_9 = \sigma_y^2[0.5(\lambda_m-\lambda_n)(0.75\mu_{040}-\mu_{220}-0.25)+1]$ and $d_9 = \sigma_y^2\sqrt{[\lambda_m(\mu_{400}-1)+(\lambda_m-\lambda_n)\{0.25(\mu_{040}-1)-(\mu_{220}-1)\}]}$	
$V_{10} = s_y^2\exp\left(\frac{s_x^2-s_x'^2}{s_x^2+s_x'^2}\right)$	$g_{10} = \sigma_y^2[0.5(\lambda_m-\lambda_n)(\mu_{220}-0.25\mu_{040}-0.75)+1]$ and $d_{10} = \sigma_y^2\sqrt{[\lambda_m(\mu_{400}-1)+(\lambda_m-\lambda_n)\{0.25(\mu_{040}-1)+(\mu_{220}-1)\}]}$	
$V_{11} = s_y^2\left[W\exp\left(\frac{s_x'^2-s_x^2}{s_x'^2+s_x^2}\right)+(1-W)\exp\left(\frac{s_z^2-s_z'^2}{s_z^2+s_z'^2}\right)\right]$	$g_{11} = \sigma_y^2[0.5(\lambda_m-\lambda_n)W(0.25\mu_{040}-\mu_{220}+0.75) -0.5(\lambda_m-\lambda_n)(1-W)(0.25\mu_{004}-\mu_{202}+0.75)+1]$ and $d_{11} = \sigma_y^2\sqrt{\left[\begin{matrix}\lambda_m(\mu_{400}-1)+(\lambda_m-\lambda_n)\{0.25W^2(\mu_{040}-1)\}\\+0.25(1-W)^2(\mu_{004}-1)-W(\mu_{220}-1)\\+(1-W)(\mu_{202}-1)-0.5W(1-W)(\mu_{022}-1)\}\end{matrix}\right]}$ where $W = \frac{\mu_{004}+2(\mu_{220}+\mu_{202})+\mu_{022}-6}{\mu_{040}+\mu_{004}+2\mu_{022}-4}$	