Int. J. Production Economics 143 (2013) 275–284

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Optimal time-based consolidation policy with price sensitive demand

Ki-sung Hong [a], Chulung Lee [b,*]

[a] Graduate School of Information Management and Security, Korea University, Anamdong 5-ga, Seongbuk-gu, Seoul 136-713, Republic of Korea
[b] Division of Industrial Management Engineering and Graduate School of Management of Technology, Korea University, Anamdong 5-ga, Seongbuk-gu, Seoul 136-713, Republic of Korea

ARTICLE INFO

Available online 19 June 2012

Keywords:
Pricing
Shipment consolidation
Time-based consolidation policy
Linear demand function

ABSTRACT

We consider a single-item inventory system where shipments are consolidated to reduce the transportation cost using a time-based consolidation policy and develop a mathematical model to obtain the optimal price, replenishment quantity and dispatch cycle to maximize the total profit. The long-run average profit is computed and the optimality properties are obtained. Using the optimality properties, we develop an efficient algorithm to obtain the optimal values of price, replenishment quantity and dispatch cycle for the proposed policy. We then extend our results to consider quantity discount of unit dispatch cost. In order to compare the performances of the proposed policy with the optimal quantity-based policy, extensive numerical experiments are conducted, and the total profit and the customer waiting time that represents quality of service are compared. We also propose the quantity–time-based policy, which is a hybrid approach, and the numerical results show that additional profit can be obtained without sacrificing quality of service.

1. Introduction

In a global market, an emphasis on supply chain coordination has recently increased (Cetinkaya and Lee, 2000). If no coordination exists, the supply chain parties act independently, and this independent decision usually cannot ensure that the parties as a whole reach the optimal state (Sajadieh and Jokar, 2009). In keeping with this trend, we focus on the coordination efforts aimed at the integration of pricing, inventory and transportation.

Previous integrated supply chain optimization models are classified by two different approaches. One of the approaches is the coordination of inventory replenishment and shipment scheduling. The practice that several small orders are consolidated into a larger order is known as shipment consolidation (Chen et al., 2005; Cetinkaya et al., 2006). The main motivation of shipment consolidation is to take advantage of the decreased per unit dispatch costs due to transportation economies of scale. However, shipment consolidation increases the inventory holding cost since the inventory level increases while small orders are consolidated. Thus, inventory replenishment and shipment consolidation must be optimized jointly.

In transport logistics, two different types of shipment consolidation policies (a time-based policy and a quantity-based policy) are employed (Cetinkaya et al., 2006). A time-based policy dispatches accumulated orders every T period whereas a quantity-based policy dispatches accumulated orders to reach a predetermined economic dispatch quantity. A time-based policy was studied by Cetinkaya and Lee (2000, 2002) and Moon et al. (2011). Cetinkaya and Lee (2000) develop a renewal theoretical model for the case of Poisson demands, and compute the optimal replenishment quantity and dispatch frequency simultaneously. Cetinkaya and Lee (2002) present an optimization model to determine the optimal freight consolidation cycle and the optimal number of dispatch decision within an inventory replenishment cycle. Moon et al. (2011) extend the results of Cetinkaya and Lee (2002) to consider the multiple items, and they developed two joint replenishment and consolidated freight delivery policies for a TPW (third party warehouses).

Cetinkaya and Bookbinder (2003) also apply renewal theory to two shipment consolidation policy. For the case of a quantity-based policy, they obtain the optimal target weight before dispatch, while for a time-based policy, they calculate the optimal length of each consolidation cycle. Chen et al. (2005) and Cetinkaya et al. (2006) present two models (quantity-based and time-based) for joint stock replenishment and shipment consolidation which arise in the context of vendor managed inventory. Cetinkaya et al. (2006) analyze the advantages and disadvantages of the quantity-based policy and the time-based policy, and propose a hybrid policy (quantity–time-based policy). Mutlu et al. (2010) present an analytical model for computing the optimal hybrid policy parameters. However, they assume that price is fixed, and their model did not allow the product price change. Our study differs from these models in that we allow the product price change, and obtain the optimal pricing policy.

The other supply chain optimization problem is the integration of inventory and pricing decisions. The integration of inventory

* Corresponding author. Tel.: +82 2 3290 3395; fax: +82 2 929 5888.
E-mail address: leecu@korea.ac.kr (C. Lee).

0925-5273/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ijpe.2012.06.008

and pricing problem was first investigated by Whitin (1955), who incorporate pricing into the traditional EOQ model through a linear price-sensitive relation for the customers. Other researchers such as Mills (1959) and Polatoglu (1991) also investigated a single-period "Newsboy" model to consider linear demand function. Abad (1996) then investigated a similar problem for a more general demand function. For stochastic demand, Federgruen and Heching (1999) address the problem of determining optimal pricing and inventory replenishment quantities. They build both finite and infinite horizons and obtain an optimal combined pricing and inventory policy. Lau and Lau (2003) investigate a joint pricing-inventory model and they found that the nature of the price–demand relationship may have a considerable effect on the results of inventory-pricing decision. Abad (2008) investigated the pricing and lot-sizing problem under a condition of partial backordering. Recently, Chen and Yang (2010) studied a single product, periodic review inventory model, in which pricing and ordering decisions are made simultaneously over finite time horizon. However, these models did not consider outbound shipment scheduling. Our study differs from these models in that we consider shipment consolidation, and obtain the optimal shipment policy.

Recently, Ulku and Bookbinder (2010) investigate the effects of different pricing schemes for a Third Party Logistics (3PL) provider who tenders a consolidated load to a carrier. They present an optimization model for integrating pricing and transportation decisions, and derive the optimal quotations that should be made for price and delivery time with the objective of maximizing the profit. However, their model considers only the unit shipping cost and consolidation penalty cost, and did not consider inventory replenishment. Our study differs from Ulku and Bookbinder (2010)'s model in that we consider inventory replenishment cost, and determine the optimal ordering policy.

In this paper, we consider a single-item inventory system where shipments are consolidated to reduce the transportation cost, and we develop an optimization model where the shipment, ordering and pricing policies are optimized all together.

This paper is organized as follows. In Section 2, we develop a mathematical model to maximize the total profit with price sensitive demand, and an efficient algorithm is provided to obtain the optimal values for the proposed policy. We then extend our results to consider quantity discount of unit dispatch cost. In addition, we also propose a quantity–time-based policy. In order to compare the performances of the proposed policy with the optimal quantity-based policy, extensive numerical experiments are conducted, and the total profit and the customer waiting time that represents quantity of service are compared in Section 3. In addition, we also compare the profit and quality of service of the quantity–time-based policy with those of the optimal time-based policy.

2. Optimal time-based consolidation policy

2.1. Assumptions and notations

In this study, we consider a system operated by a time-based policy. The time between two successive dispatch decisions, T, is called a dispatch cycle. In order to employ a shipment consolidation, customers are willing to wait (i.e., backordering) and all demand must be satisfied. Thus, if backordering is not allowed, a shipment consolidation cannot be implemented. We assume that the customer order fulfillment may be postponed during a dispatch cycle. However, the postponement of order may result in customer waiting (i.e., backordering). This postponement has negative impact on customer's goodwill. Such loss of goodwill associated with delayed receipt of goods is represented by a customer waiting cost and this cost can be interpreted as the backorder cost (Chen et al., 2005). Axsater (2006) defined that the backorder cost is a certain penalty cost proportional to the waiting time for the customer. In this paper, we employed the waiting time cost per unit per unit time. In general, three types of backorder costs are commonly considered in inventory literature (Hu et al. 2009). One of these is constant for each unit backordered (per unit). Another type is proportional to the waiting time (per unit). The final type of backorder costs accumulates at a constant rate proportional to both the backorder volume and the waiting time (per unit per unit time). Thus, the third type of backorder costs may be considered the waiting time cost. The customer demand rate λ is assumed to be a linear function of product price, p, i.e., $\lambda = a - bp$, where $a>0$ and $b>0$. This demand function has been commonly employed extensively in literature relating to pricing and inventory problems (Whitin, 1955; Mills, 1959; Polatoglu, 1991; Abad, 1996, 2008; Lau and Lau, 2003; Chew et al., 2009 etc.).

The followings are the additional assumptions of the model:

the inventory level is under periodic review;
the demand arrival rate λ depends linearly on price,
the lead time for inventory replenishment is assumed to be negligible.

Fig. 1 shows the vendor's inventory level. Let Q denote the replenishment quantity and K is an integer denoting the number of dispatch cycles within a replenishment cycle. Under the time-based policy, a new dispatch cycle is started every T time units. In turn, a replenishment cycle includes at least one dispatch cycle. Inventory is replenished when the cumulative demand exceeds on-hand inventory. Let K denote the number of dispatch cycles within a given inventory replenishment cycle; K is computed by

$$K = \inf\left\{ k : \sum_{i=1}^{k} D_i(T) > Q \right\}$$

where $D_i(T)$denote the demand during dispatch cycles, $i = 1,\ldots,K$.

The following notations are employed in this study:

p	: unit product price (decision variable)
F_R	: fixed cost of inventory replenishment
C_R	: incremental cost of unit inventory replenishment
F_D	: fixed cost of customer order shipment
C_D	: incremental cost of customer order shipment
h	: holding cost per unit per unit time
w	: waiting cost per unit per unit time
T	: dispatch cycle time (decision variable)
K	:number of dispatch cycles within a replenishment cycle
Q	: inventory replenishment quantity (integer, decision variable)

The objective is to develop an optimization model to jointly determine the optimal price, p, the optimal replenishment quantity, Q, and the optimal dispatch cycle, T, in order to maximize the total profit.

2.2. Mathematical model

In this section, we present a mathematical model for optimal pricing and inventory management in a time-based dispatch system. The long-run average profit, $TP(p, T, Q)$, is determined by dividing E[replenishment cycle profit] by E[replenishment cycle length].