Int. J. Production Economics 143 (2013) 335–341

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

# Analysis of an assemble-to-order system with different review periods

A.G. Karaarslan [a,*], G.P. Kiesmüller [b], A.G. de Kok [a]

[a] Technische Universiteit Eindhoven, Postbus 513, NL-5600 MB Eindhoven, Netherlands
[b] Christian-Albrechts-Universität zu Kiel, Institut für Betriebswirtschaftslehre, Olshausenstraße 40, D-24098 Kiel, Germany

ARTICLE INFO

Available online 2 December 2011

*Keywords:*
Inventory management
Assemble-to-order system
Base stock policy
Stochastic demand
Fixed replenishment intervals

ABSTRACT

We consider a single item assembled from two components. One of the components has a long lead time, high holding cost and short review period as compared to the other one. We assume that net stocks are reviewed periodically, customer demand is stochastic and unsatisfied demand is backordered. We analyze the system under two different policies and show how to determine the policy parameters that minimize average holding and backorder costs. First, we consider a pure base stock policy, where orders for each component are placed such that the inventory position is raised up to a given base stock level. In contrast to this, only the orders for one component follow this logic while the other orders are synchronized in case of a balanced base stock policy. Through mathematical analysis, we come up with the exact long-run average cost function and we show the optimality conditions for both policies. In a numerical study the policies are compared.



## 1. Introduction

In real-life supply chains, individual items have their own lot sizing and lead time constraints based on contracts with suppliers or production process characteristics. Coordination of release decisions across multiple items is thereby not an easy task. In the existing literature, convenient assumptions are made, such as equal lot sizes for items (e.g. Svoronos and Zipkin, 1988), equal review periods (e.g. Clark and Scarf, 1960), nested lot sizes (e.g. Chen, 2000) and nested review periods (e.g. Van Houtum et al., 2007). One of the consequences of these assumptions is that upstream items and long-lead-time items should have larger lot sizes. Unfortunately, in practice, simple and cheap materials can have short lead times whereas complex and expensive materials usually have long lead times. On top of that the economic order quantity of complex and expensive items implies that such items should be ordered more frequently than cheap items if they have equal demand rates.

Whether an item is cheap or expensive is generally determined by the complexity and capital intensity of the production processes. Complex processes consisting of multiple transformation steps require longer lead times. On the other hand, capital-intense production is characterized by high utilization, which naturally translates into long lead times. Thus, in practice long-lead-time items are often more expensive than short-lead-time items. Typical examples of this situation can be observed in high volume electronics and pharmaceutics industry, where key components (e.g. LED screens, active ingredients) have lead times beyond 10 weeks, whereas cheap components (e.g. plastic parts, packaging material) have lead times of less than several weeks. Similarly, in capital goods industry, where typically products are assembled to order, expensive items (e.g. magnets for medical scanning equipment, lenses for lithography machines) are ordered daily or weekly, while metal and plastic parts may be ordered monthly on average. Such lead time and review period relations between components also exist in make-to-order and configure-to-order environments.

In this context, we consider a two component assemble-to-order (ATO) system, where the inventory levels are reviewed periodically. One component has a high holding cost, long lead time and short review period, whereas the other component has a relatively low holding cost, short lead time and long review period. We further assume that lead times are deterministic and review periods are determined exogenously. Customer demand is stochastic and unsatisfied customer demand is backlogged. The objective is to minimize the expected cost per period consisting of holding and backordering costs by determining the optimal policy parameters. Since the form of the optimal policy is not known for this system, we explore the performance of two different heuristic inventory control policies and determine the cost optimal policy parameters minimizing holding and backorder costs.

The first policy considered is the pure base stock policy in which replenishment orders are placed to restore a fixed base stock level for each component. This policy is well studied in the literature on ATO systems and widely applied in practice. Under a periodic review setting, pure base stock policies are shown to be

* Corresponding author. Tel.: +31 40 2473828; fax: +31 40 2474985.
*E-mail address:* a.g.karaarslan@tue.nl (A.G. Karaarslan).

0925-5273/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2011.11.030

optimal for serial systems with equal review intervals by Clark and Scarf (1960) and with nested review intervals by Van Houtum et al. (2007). Rosling (1989) shows that the results and methods for serial systems can be used for solving pure assembly systems. However, our problem does not fit into any of these cases due to the review period constraints. If we apply Rosling's (1989) approach to our model, the equivalent serial network does not have the required nested review intervals property. As a consequence, Van Houtum et al.'s (2007) result cannot be applied to this model.

The second inventory control policy we consider is the so-called "balanced base stock policy". Here, we assume a base stock policy for the longest lead time component. Then, all other components' base stock levels are coordinated with respect to the stock level of this pivot component. Balanced base stock policy was first studied by Zhang (1995) for an ATO system with one end-item and equal replenishment intervals. The analytical results show that indeed the system behaves like a single stockpoint.

In ATO systems, there are two major challenging problems. The first one is the component allocation problem for the case of multiple end-items. As we study a single end-item model, this problem does not occur. The second problem is minimizing the expected number of backorders or item-based backorders under pure base stock policies. In general, this is computationally demanding because the process involves joint probabilities and optimization of nonseparable functions.

The literature on discrete-time ATO systems considers both of these issues. Hausman et al. (1998) study an ATO system with a decentralized base stock policy and normal distributed demand. They propose an equal fractile method for non-stockout probability and develop a heuristic for maximizing a lowerbound on the order fill rate. Zhang (1997) and Agrawal and Cohen (2001) study a similar system where the objective is to minimize total inventory investment subject to a service level constraint. Zhang (1997) defines a so-called fixed-priority allocation rule and concentrates on demand fulfillment rates. On the other hand, Agrawal and Cohen (2001) gain managerial insights on the problem when the component allocation is based on fair-shares rule. Akçay and Xu (2004) introduce a simple and order-based component allocation rule and compare it with the previously stated ones. De Kok (2003) defines a set of ATO systems named as "strongly ideal". Then, through rigorous analysis he finds exact expressions for the performance characteristics of such systems and develops efficient approximation methods.

In continuous-time framework, most research focuses on computing order-based backorders, performance measures like order-fulfillment rates or finding bounds for item-based backorders. The most recent work in this setting includes Song (2002), Song and Yao (2002), Lu et al. (2005), Lu and Song (2005) and Hoen et al. (2011). All these papers assume ATO systems with Poisson distributed customer demand and pure base stock policies. Finally, we refer the reader to the book chapter of Song and Zipkin (2004) for an extensive literature review on ATO systems.

To the best of our knowledge, this is the first paper on ATO systems with different review intervals for each component. We compute the exact expressions for expected cost per period, and we derive optimality conditions. Reinforcing the numerical results of De Kok (2003), we analytically prove the equivalence of non-stockout probability to newsboy fraction at optimality for both pure base stock policy and balanced base stock policy.

The remainder of the article is organized as follows. Firstly, we describe the detailed model assumptions and the related total cost function in Section 2. Secondly, we formulate and analyze the optimization models based on pure base stock policy and balanced base stock policy in Section 3. Next, in Section 4, we present numerical results to asses and compare the system performance under both replenishment policies. Finally, we summarize the main contributions of this study and give directions for further research in Section 5.

## 2. The assemble-to-order model

We have a single item that is assembled from two components. One piece of each component is needed to produce one end item. The expensive component is stocked at stockpoint 1 and the cheap component is stocked at stockpoint 2. It is assumed that the inventories of components are replenished from suppliers with infinite capacity. Whenever customer demand occurs, the end item is assembled immediately if both components are available otherwise it is backordered.

Time is divided into periods of equal length and the planning horizon is infinite. We want to make a clear distinction between a "period" and a "review period". Without loss of generality each period is assumed to have length 1 and periods are numbered as $\{0,1,2,\ldots\}$. A review period, on the other hand, is composed of multiple periods where at the beginning of a review period the stock levels are reviewed and orders are placed.

There are four main events that may occur during a period: (i) arrival of orders (if scheduled to this period), (ii) placing of orders (if the period is also the beginning of a review period), (iii) occurrence of demand, (iv) incurring costs. The first three events take place at the beginning of the period. We assume that customer demand occurs after ordering decisions are made. Holding and penalty costs are incurred at the end of each period.

We define $I_n(t)$ as the total on-hand inventory of component $n$ at the end of period $t$. The net stock of a component equals all on-hand inventory at this stockpoint minus the amount of backorders. $X_n(t)$ denotes the net stock of component $n$ at the end of period $t$. Also, we define the inventory position of a component as its net stock plus all material in transfer to that stockpoint. Let $IP_n(t)$ be the inventory position for component $n$ at the beginning of a period $t$ after ordering decision is made.

Component $n$ has a review period of length $R_n$ such that the inventory position of $n$ is reviewed and replenishments are made every $R_n$ periods. We assume that component 2's review period is an integer multiple of component 1's review period. Further, we define $r \in \mathbb{N}$ as the number of times that component 1 can be ordered per order of component 2. Thus, the relationship is $R_2 = rR_1$ and $R_2 \geq R_1$ by definition.

Customer demand in each period is independent and identically distributed with density function $f(\cdot)$ and distribution function $F(\cdot)$. $D[t,t+1)$ represents the demand during period $t$ with expected value $\mu$, variance $\sigma^2$, and coefficient of variation $c_\nu$. Cumulative demand occurring during a time interval between the beginning of period $t_1$ and till the beginning of period $t_2$ $(0 \leq t_1 < t_2)$ is denoted by $D[t_1,t_2)$. Further, we assume that $F(D[t,t+1) < 0) = 0$.

The lead time $L_n$ between placing and arrival of an order for stockpoint $n$ is assumed to be deterministic and it is defined in periods. The relation between the lead time of the components is $L_2 < L_1$.

We further assume synchronization in the timing of order arrivals such that an order arrival at stockpoint 2 always coincides with an order arrival at stockpoint 1. Without loss of generality, we assume that stockpoint 1 places an order at the beginning of period zero. Thus, the ordering periods of stockpoint 1 are defined by the set $T_1 = \{kR_1 | k \in \mathbb{N}_0\}$. An order placed by stockpoint 1 at period $t$ $(t \in T_1)$ will arrive at the beginning of period $t+L_1$ in which an order of stockpoint 2 will also arrive. Since $R_2$ is an integer multiple of $R_1$, at periods $kR_2+L_1$ (where $k \in \mathbb{N}_0$) there will be an arrival of both components. So, the set of ordering periods