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Inventory control in supply chains: Alternative approaches to a two-stage lot-sizing problem

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ABSTRACT

The principal challenge of inventory control in supply chains is that the interacting autonomous enterprises have to plan their production and logistics under information asymmetry, driven by different, often conflicting objectives. In this paper, four different computational approaches are investigated to cope with this challenge: decomposition, integration, coordination, and bilevel programming. The four approaches are applied to solving the same two-stage economic lot-sizing problem, and compared in computational experiments. The prerequisites of the approaches are analyzed, and it is shown that the profits realized and the costs incurred at the different parties largely depend on the solution approach applied. This research also resulted in a novel coordination mechanism, as well as a new algorithm for the bilevel optimization approach to the investigated lot-sizing problem. A specific goal of this study is to highlight the so far less recognized application potential of the coordination and the bilevel optimization approaches for controlling inventories in a supply chain.

1. Introduction

The principal challenge of inventory control in supply chains is that the autonomous enterprises have to plan their production and logistics under information asymmetry, driven by different, often conflicting objectives. Moreover, the individual enterprises typically make decisions that affect the entire supply chain, and for this purpose they also exploit private information that is inaccessible to the other parties.

This paper investigates four different approaches to cope with this challenge. According to the classical *decomposition approach*, each party optimizes its own production and logistic decisions without explicitly considering the consequences on the supply chain level. The *integrated approach* optimizes the overall performance of the supply chain by centralized planning, however, this requires a tight integration of the parties. By lifting the latter requirement, the *coordinated approach* seeks for mechanisms that motivate the autonomous enterprises to cooperate in finding mutually beneficial plans by negotiation and benefit sharing. Finally, the *bilevel approach* enables an individual party, in possession of sufficient information about its partners, to optimize its production taking into account the actions that it can expect from the partners.

The goal of this study is to provide a clear-cut comparison of the above fundamental approaches by applying them to a common problem model. The main modeling, computational, and managerial implications are investigated with a focus on the prerequisites of each approach, such as the availability of information, the contractual requirements, or the assumptions on the type of cooperation. Furthermore, the potential gains for the different parties of adopting a given approach are examined, and the resulting solutions, profits and costs are compared. To the best of our knowledge, this is the first study that provides a self-contained comparison of these approaches, applied to the same inventory control problem in different settings. A specific goal of the paper is to highlight the benefits of the two less recognized approaches, coordination and bilevel optimization, for the different parties in the supply chain. A new coordination mechanism (Section 5) and a new algorithm for solving the bilevel version of the investigated lot-sizing problem (Section 6) are also presented.

The investigated problem corresponds to an uncapacitated economic lot-sizing problem in a two-echelon supply chain. In a dyadic situation where a *buyer–supplier* chain meets external demand, this problem involves both the production related decisions of the supplier, as well as the logistic decisions of the buyer. Although for the sake of analytical clarity some simplifying assumptions have to be taken, the basic problem has direct

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application relevance. Primarily, a retailer may assume the role of the buyer, connecting exogenous market demand and the service of the supplier. Further on, a similar buyer–supplier relationship may hold between multiple divisions of a large enterprise.

For a review of inventory control problems, both as faced by a single decision maker and in a supply chain, the reader is referred to Axsäter (2006). The potential gain by integrated versus decentralized decision making in supply chains is investigated in Perakis and Roels (2007), where the difference of the induced costs is defined as the *price of anarchy*. The coordination of supply chains consisting of autonomous enterprises is studied in detail in Albrecht (2010), while a comprehensive taxonomic survey of coordinated buyer–vendor models in a deterministic, time invariant setting is provided in Sarmah et al. (2006). The fundamental ideas of bilevel programming are presented in Dempe (2002), and the application of this approach to the management of multi-divisional organizations is studied by Bard (1983). Further, more specific references are provided later in Sections 3–6, each of which investigate one of the four possible computational approaches to the studied lot-sizing problem.

2. Problem definition

2.1. A two-stage lot sizing problem

The different computational approaches are studied on a two-stage single-item uncapacitated lot-sizing problem as follows. Let us consider a supply chain that provides a single item to its customers. The supply chain consists of two independent companies, a *buyer* and a *supplier*. The buyer (and hence, the supply chain) faces dynamic, deterministic external demand d_t, $t=1,\dots,T$, over a discrete time horizon of T time periods.

Departing from the known demand, the buyer computes its *supply requests*, i.e., the amount x_t^1 of the item that should be delivered from the supplier to the buyer in each time period t. The buyer may use the delivered amount partly to satisfy the demand in the same period t, partly to keep it on stock to cover future demand in periods $t' > t$, and partly to satisfy backlogged demand from previous periods $t'' < t$. Delivering a positive amount in period t incurs a fixed cost of f_t^1 plus a per unit cost of p_t^1. Holding inventory and backlogging at the buyer take h_t^1 and g_t^1 per unit and per period cost, respectively. These delivery, holding, and backlogging costs are paid by the buyer to an external party.

The income of the buyer consists of the per unit purchase price q_t^1. Symmetrically, the buyer pays a per unit purchase price q_t^2 for the ordered goods. This purchase price is independent of the above logistic costs.

To cover the demand set by the buyer's supply requests, the supplier generates a *production plan* that specifies the amount x_t^2 of the item to be produced in period t over the planning horizon. In each period t where a positive amount $x_t^2 > 0$ is produced, production cost is incurred: a fixed setup cost of f_t^2 plus a per unit cost of p_t^2. Just as the buyer, the supplier can hold stock or backlog demand, for a cost of h_t^2 and g_t^2 per unit and per period, respectively. Moreover, it is assumed that the production and holding costs that occur at the supplier are paid by the supplier to an external party, whereas the backlogging cost is paid by the supplier to the buyer as a penalty for the delay caused.

Furthermore, it is assumed that all demand must be satisfied by the end of the horizon and no item remains in stock, i.e., $\sum_{t=1}^T d_t = \sum_{t=1}^T x_t^1 = \sum_{t=1}^T x_t^2$. The production and delivery lead times are zero. The objective of both parties is to maximize their profits.

In all models studied in the sequel the decision variables of the buyer are the x_t^1 supply, s_t^1 inventory and r_t^1 backlog quantities for each time period $t=1,\dots,T$ of the planning horizon. The supplier has a decision problem of identical structure, with x_t^2 production, s_t^2 inventory and r_t^2 backlog quantities. Whenever appropriate, we distinguish the two parties with an upper index k, where $k=1$ stands for the buyer's and $k=2$ for the supplier's decision variables and parameters. Auxiliary binary variables y_t^1 and y_t^2 are introduced to capture events of delivery and production, respectively. The notation is summarized in Table 1.

2.2. Plans and realization

Since the above model allows the supplier to backlog, according to some of the investigated approaches, the buyer may not be able to anticipate situations where the realized deliveries from the supplier deviate from the supply requests. Therefore, the executed scenario may differ from the plan, and the rules of the execution must be established. The following rules are applied.

If the supplier produces the goods on time, then the buyer must call off the amount indicated in the supply requests. Otherwise, i.e., if the supplier backlogs demand, then the buyer calls off the ordered goods as soon as they are available. Formally, in each period t, the buyer must call off the amount that has been

Table 1
The notation used in the paper.

Dimensions	
T	Number of time periods
Upper indices	
$\square^1$	Parameters/variables related to the buyer (planned values)
$\square^{1R}$	Parameters/variables related to the buyer (realized values)
$\square^2$	Parameters/variables related to the supplier (plans match realization)
Parameters	
d_t	External demand in period t
f_t^k	Fixed delivery ($k=1$)/production ($k=2$) cost in period t
p_t^k	Per unit delivery ($k=1$)/production ($k=2$) cost in period t
h_t^k	Per unit and per period holding cost at party k in period t
g_t^k	Per unit and per period backlog cost at party k in period t
q_t^k	Per unit purchase price at party k in period t
Variables	
x_t^k	Amount of goods requested by the buyer ($k=1$)/produced by the supplier ($k=2$) in period t
y_t^k	Binary variable indicating whether a positive amount is produced/delivered in period t
s_t^k	Stock at party k at the end of period t
r_t^k	Backlog at party k at the end of period t
Performance measures	
C^k	Total production and logistic cost incurred at party k
P^k	Profit realized by party k