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Risk preferences of a newsvendor with service and loss constraints

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ABSTRACT

Using the newsvendor framework, we present an approach to determine an order quantity, which is relevant for the observed ordering behavior of an inventory manager. Whereas in literature just one constraint has been used, we suggest two conflicting constraints. The service constraint specified by a target value for the cycle service level is fulfilled by high order quantities, the loss constraint is characterized by a target resulting in losses is met by low order quantities. Choosing as objective function a mean-deviation rule where risk attitudes are measured by the conditional value at risk the optimal order quantity is determined depending on the risk parameters. If the newsvendor is not able to specify the risk preferences we use the constraints to describe the risk attitudes of the decision maker in dependence of the profitability of the product and its demand distribution including those products where no admissible order quantity exists. If there are admissible solutions we identify conditions such that the service constraint is dominating, i.e. the prescribed service level is high and necessarily risk taking behavior is implied. Contrary, if the loss constraint dominates, i.e. the probability of loss is low the decision maker in any case is a risk averter. But if both the service target and the loss target are not too challenging then for products with high profitability all risk preferences are possible.

1. Introduction

Experimental findings state that the actual order quantity deviates from the optimal quantity prescribed by the newsvendor model maximizing the expected profit. Moreover empirical observations show that managers tend to order less than the classical newsvendor because they base their decisions on other performance measures besides expected profit (Brown and Tang, 2006; Schweitzer and Cachon, 2000). From the perspective of supply chain management an inventory manager should control not only internal-facing performance measures like expected profit and the probability of loss but also customer-related measures like the level of product availability. E.g. an incentive to order less than the expected profit maximizing quantity is to control the probability of loss. An incentive to order more may be to achieve a high service level resulting in greater customer satisfaction.

Using the newsvendor framework, we present an approach to determine an order quantity, which is more relevant for the observed ordering behavior of an inventory manager. A comprehensive survey of extensions of the classical single-period inventory model is contained in Khouja (1999); for a recent review we refer to Qin et al. (2011). The proposed model combines two important streams of the newsvendor framework. We consider a model where the decision maker does not express risk-neutral behavior, i.e. the objective function is not the expected profit. Eeckhoudt et al. (1995) present a newsvendor in the expected utility framework. They demonstrate that a risk-averse decision maker orders less than the classical newsvendor. Other authors propose to use the conditional value at risk (CVaR) as objective function (see e.g. Chiu and Choi, 2010; Jammernegg and Kischka, 2007 and the references contained therein). The ordering behavior is in accordance with that of the expected utility approach. It will be explained in more detail in the next section. Historically the first alternative objective function in the newsvendor setting is the maximization of the probability to exceed a specified profit target (Lau, 1980). There are a number of generalizations: Parlar and Weng (2003) use a moving target – they propose the expected profit – which depends on the order quantity. In a recent paper Yang et al. (2011) analyze a price-setting newsvendor under both profit and revenue targets.

The second extension of the classical newsvendor model that we build on is the consideration of constraints. A group of papers suggests loss-averse constraints like the CVaR or the value at risk (VaR) (see e.g. Özler et al., 2009; Zhang et al., 2009). The VaR constraint specifies that the probability of low profits and thus also losses must not exceed an upper bound, i.e. it is just the

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opposite of the profit target previously discussed as an objective function. Other papers consider a service constraint. Chen and Chuang (2000) specify a lower bound for the fill rate; Sethi et al. (2007) do so for the in-stock probability, which we call cycle service level. There is a correspondence between a service constraint and the revenue target, which is used as an objective function because a higher order quantity in general means an increase of the level of product availability and also an increase of revenues.

With this brief review of the related literature in mind we can state three sets of contributions of the paper:

- Whereas in literature up to now just one type of constraint has been used, we suggest two conflicting constraints. The service constraint specified by a lower bound for the cycle service level will be fulfilled for high order quantities, the loss constraint is characterized by an upper bound for resulting in losses and will be met by low order quantities. There is a rich literature documenting the importance of achieving revenue as well as profit targets (see Yang et al., 2011; and the references contained therein).
- We start the analysis of the newsvendor model with constraints by choosing as objective function a mean-deviation rule where risk is measured by CVaR. This is done to determine the optimal order quantity in dependence of the risk parameters and its structure, which turns out to be a two-sided control limit policy. This structure of the optimal order quantity was proved by Jammernegg and Kischka (2008) for the expected profit as objective function. There the focus is on the impact of demand variability and in addition the price-setting newsvendor is analyzed.
- If the newsvendor cannot specify the risk preferences we use the constraints to describe the risk attitudes of the decision maker in dependence of the profitability of the product and its demand distribution including the range of products where no admissible solution exists, i.e. there is no order quantity fulfilling both constraints. Typically, for products with high profit value all risk preferences for the decision maker turn out to be possible. Otherwise, the prescribed values of the performance measures can be attached to specific risk attitudes.

After introducing the notation using the classical newsvendor model we state the optimal order quantity depending on the risk parameters for the unconstrained newsvendor model with mean-deviation rule as objective function. Then we derive the set of admissible order quantities for a service constraint – a lower bound for the cycle service level – and for a loss constraint specified by an upper bound for the probability of loss. Using the optimal order quantity of the newsvendor model with risk preferences we analyze the implication of the relations of the profit value of the product and the prescribed performance measures. For given risk preferences we present conditions, which imply that the optimal admissible solution is a corner solution.

Finally, this enables us to derive implications from the prescribed service and loss constraints for the risk preference of the decision maker. Assuming that the optimal unrestricted solution is an admissible order quantity one can characterize the risk preferences of the newsvendor. If the cycle service level is larger than the profit value an admissible solution exists if the prescribed probability of loss is high enough. In this case the decision maker is necessarily a risk taker. On the other hand it is not necessary for a risk taker that the prescribed cycle service level is larger than the profit value; this is the case if the loss constraint is dominating. Similar considerations can be made for risk aversion. These results depend crucially on the demand distribution.

2. Newsvendor with risk preferences

First we introduce our notation for the newsvendor model. X denotes the random demand with distribution function F; we always assume that F is continuous and strictly monotone increasing. The purchase price per unit is c, the selling price per unit is p. Unsatisfied demand is lost and leftover inventory of the product is sold at the salvage value per unit z. We assume $p>c>z$.

For the order quantity y the random profit is denoted by $g(y,X)$. With $(y-X)^+=\max(0,y-X)$ we have

$$g(y,X)=(p-c)y-(p-z)(y-X)^+ \tag{1}$$

As mentioned in our introduction there are many different ways to define an optimal order quantity. The classical solution maximizes the expected value of (1).

Let

$$pv=\frac{p-c}{p-z} \tag{2}$$

denote the profit value of the product. Then the classical solution is given by

$$y^*=F^{-1}(pv) \tag{3}$$

We now shortly describe our preference functional (see Jammernegg and Kischka, 2007). For each $\alpha\in\,]0,1[$ denote by $z_\alpha(y)$ the α-quantile of the profit $g(y,X)$; α is chosen by the decision maker as a borderline between sufficiently good profits exceeding $z_\alpha(y)$ and disappointing profits below $z_\alpha(y)$. Besides α the decision maker selects – similar to the well-known Hurwicz–criterion – a coefficient of pessimism $\lambda\in\,]0,1[$. The preference functional $\Phi(g(y,X))$ is given by

$$\Phi(g(y,X))=\lambda E(g(y,X)|g(y,X)\le z_\alpha(y)) \\ +(1-\lambda)E(g(y,X)|g(y,X)\ge z_\alpha(y)) \tag{4}$$

i.e. the order quantity y is evaluated by the expected value of disappointing profits weighted by the factor λ and by the expected value of sufficiently good profits weighted by $1-\lambda$.

The first conditional expected value in (4) is the so called conditional value at risk ($CVaR_\alpha$) of the profit. Since

$$E(g(y,X))=\alpha E(g(y,X)|g(y,X)\le z_\alpha(y) \\ +(1-\alpha)E(g(y,X)|g(y,X)\ge z_\alpha(y))$$

we can rewrite (4) as a mean deviation rule, where risk is measured by the conditional value at risk:

$$\Phi(g(y,X))=\frac{\lambda-\alpha}{1-\alpha}CVaR_\alpha(g(y,X))+\frac{1-\lambda}{1-\alpha}E(g(y,X)) \tag{5}$$

For $\lambda=\alpha$ the preference functional (5) reduces to the well-known expected value criterion and therefore represents risk neutrality.

A decision maker is risk averse if the expected value of a random variable is preferred to the random variable itself and vice versa (e.g. Cohen, 1995). It can be shown that for $\alpha<\lambda$ the expected value $E(g(y,X))$ of the profit is preferred to the random variable $g(y,X)$, thus describing risk aversion. For $\alpha>\lambda$ (5) represents risk taking behavior. So we have

$$\begin{array}{ll} \alpha<\lambda & \text{represent risk averse behavior} \\ \alpha>\lambda & \text{represent risk taking behavior} \\ \alpha=\lambda & \text{represent risk neutral behavior} \end{array} \tag{6}$$

Maximizing (5) gives the optimal solution

$$y^*(\alpha,\lambda)=\begin{cases} F^{-1}\left(pv+\frac{\alpha-\lambda}{1-\lambda}(1-pv)\right) & \text{for} \quad \lambda\le pv \\ F^{-1}(pv\frac{\alpha}{\lambda}) & \text{for} \quad \lambda\ge pv \end{cases} \tag{7}$$