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Aggregate constrained inventory systems with independent multi-product demand: Control practices and theoretical limitations

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ABSTRACT

In many practical problems inventory managers are confronted with aggregate constraints that result typically from limitations in workspace, workforce, investment or from targeted service levels. In this paper we discuss some multi-product inventory problems with independent items under one or multiple aggregate constraints. We analyze some recent and relevant references grouped into five categories: deterministic lead-time demand, news vendor, base-stock policy, (r, Q) policy and (s, S) policy. We investigate the proposed model formulations, the algorithmic approaches and benefits of a system approach versus an item approach. A multi-product wholesaler case study is presented. Finally we highlight the limitations from a practical viewpoint of these models and point out some possible direction for future improvements.

1. Introduction and motivation

The issues addressed in this paper are concerns and problems encountered in practice by managers who are confronted with system wide goals on service level or costs. As such the company can have for example a strategy to achieve an overall fill rate service level of 97% for this year. This service level may be part of a service contract which has a financial impact in the form of costly penalties if this pre-set target service level is not achieved. In practice managers need to find solutions for the limited available capacity of several resources. The warehouse has a limited available space that is not easily surmountable without extra costs. The money available to invest in inventory also has its boundaries and is sometimes used as a direct key performance indicator. The limited available workforce capacity can be a reason to limit the number of orders, as each order requires a set of activities: administer, perform quality control, receive and put away the goods. So inventory managers have system wide limitations (space, money or workforce) or goals (service levels or costs), while the majority of classic inventory closed formulas focus on single items and are unable or inefficient to realize these conditions.

Applying a single item approach to attain these goals is not a best practice, neither is it effective to satisfy the system's constraints. Nevertheless we see it being applied too often within companies, without realizing the loss in efficiency or in money this has as a consequence. An IT system that lacks the support for a system wide approach may however be another significant obstacle. We believe that it is unawareness of the existence of these system approaches, by a large number of managers, or the assumed insurmountable complexity of these approaches that prevents their widespread use. As a first example of the value of these system approaches, we want to refer to Sherbrooke (2004) who reports using a system approach on 1.414 spare parts resulted in a 46% reduction of inventory investment without a decrease in performance. We believe that a better understanding and insight of multi-product inventory problems with aggregate constraints should become common knowledge for the inventory manager, knowing that the first papers on these topics date back to the sixties and seventies. This will certainly help them to achieve their system goals and will have a positive impact on the key performance indicators.

An optimal policy surface, see Gardner and Dannenbring (1979), is a practical tool to deduct the optimal link between system cost and system service, while fulfilling the system constraints. An optimal policy surface can be generated for each system based on its specific characteristics. In this paper we want to provide an overview of the relevant references for the considered policies together with some insights in the algorithms used. The usefulness in practice requires the possibility of handling large data sets and easy implementation, e.g. closed form expressions or the use of familiar software packages.

Zipkin (2000) gives a broad overview of multi-product inventory management and its several aspects. An important observation is that multi-product systems and multi-location systems are fundamentally identical. We observe the following three

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categories of multi-product inventory problems: independent items with aggregate constraints, network of items and shared supply chain processes.

The first category of independent items describes problems with distinct supply and demand processes and no supply–demand links between the items. Of course when there are no links at all between the items, each item can be treated individually. This is where we introduce one or multiple aggregate constraints on the whole set of items. These constraints are not network or supply chain process related but focus on available resources (space, investment and workforce) or system result (service level and cost).

A second multi-product inventory category is a network of items with a supply–demand relationship such as: a series system, an assembly system, a distribution system, a tree system or a general system. Axsäter (2003) offers a good overview of multi-echelon serial and distribution inventory systems in supply chains. Song and Zipkin (2003) give a detailed review on the assembly-to-order systems, this is a system with last minute assembly.

Finally there is a multi-product problem category where the items share the supply chain processes themselves. Two well known problems in this area are the joint-replenishment problem and the economic lot scheduling problem (ELSP). Axsäter (2006) discusses extensively both problems. In case of joint replenishment, a group of items should be replenished jointly as much as possible due to many reasons: joint setup costs, quantity discounts or coordinated transports. The ELSP on the opposite tries to spread the cyclic schedules for a number of items with constant demand and no backordering, due to a finite production rate and a minimized holding and ordering cost.

In this paper we will focus on the first category of multi-product inventory problems with independent items. We consider several instances of this problem and the remainder of the text is organized according to the following inventory policies:

- Deterministic leadtime demand.
- Newsvendor: a single period model with a stochastic demand and penalty costs for ordering too much or too little.
- Base-stock: an (r, Q) policy with $Q=1$, this is relevant when ordering costs are negligible compared with other costs.
- (r, Q) policy: an order of size Q is placed as soon as the inventory position falls to or below the reorder point r.
- (s, S) policy: an order is placed to reach the stock maximum level S as soon as stock falls to or below reorder point s.

2. Problem setting and review of some important concepts

2.1. Problem in scope

The inventory problems we discuss in this paper can be formulated by the following equations:

$$\text{Minimize} \quad f(x)=\sum_{j=1}^{J} f_j(x_j) \tag{1a}$$

$$\text{Subject to} \quad g_n(x)=\sum_{j=1}^{J} g_{nj}(x_j)\le e_n, \quad n=1,..,N \tag{1b}$$

$$x_j \in \mathbb{R}^m \text{ or } \mathbb{Z}^m, \quad j=1,..,J,\ m=1 \text{ or } 2 \tag{1c}$$

$$x_j \ge l_j, \quad j=1,..,J,\ l_j \in \mathbb{R}^m \text{ or } \mathbb{Z}^m,\ m=1 \text{ or } 2 \tag{1d}$$

There are J different inventory items. Each item has m (1 or 2) variables with lower bound(s) l_j. The decision variable values are real but can in some cases be integer. The functions $f_j, g_{1j}, \ldots, g_{Nj}$ are defined on $\mathbb{R}^m$ or $\mathbb{Z}^m$. We will only consider items with independent demand subject to at least one aggregate constraint ($N\ge 1$). The inventory cost of these items cannot be optimized independently due to the active aggregate constraint(s) (1b).

2.2. Review of Lagrange multipliers based solution approach

The method of Lagrange multipliers is based on the fact that the gradient vector of the objective function is perpendicular to the constraints surface at an optimal point. This method is suitable for some optimization problems with equality constraints. In case inequality constraints are also involved, one needs first to determine which of these inequality constraints are binding and then add them to the equality constraints. These constraints are called the active set of constraints and this set changes during the iterative solution search. Let ζ_n be the Lagrange variables associated to the aggregate constraints and ξ_n be the Lagrange variables of the lower bounds on these decision variables. The sum of the goal function and the product of the Lagrange multipliers with the active constraints form the 'Lagrange function' $Z(x,\zeta,\xi)$ (2a). Setting the partial derivatives of the Lagrangian function equal to zero (2b) provides a necessary condition for a solution to the constrained problem (1a)–(1d), an extensive and detailed discussion of this approach can be found in Bertsekas (1996) and Bazaraa et al. (2006):

$$Z(x,\zeta,\xi)=f(x)+\sum_{n=1}^{N}\zeta_n[g_n(x)-e_n]+\sum_{j=1}^{J}\xi_j[x_j-l_j] \tag{2a}$$

$$\nabla Z=0 \tag{2b}$$

Everett (1963) points out the usefulness of Lagrange multipliers for optimization in the presence of constraints. He underlines that it is not limited only to differentiable functions. This method is specifically useful to solve allocation problems with limited resources when faced with independent activities. Patriksson (2008) gives a survey on the continuous non-linear resource allocation problem. In his paper, a rich list of applications is given, amongst which a few inventory cases. Most of available techniques are based on iteratively finding the Lagrange multiplier(s). Within each iteration the x_j values are calculated or approximated, which allows a check on the constraint validation. The challenge in the solution of (2b) lies in limiting the number of iterations and reducing the complexity to calculate x_j in each iteration in order to find the appropriate Lagrange multipliers ζ_n and ξ_n.

Note that the problem (1a)–(1d) can also be approached using other techniques than Lagrange multipliers. Some authors apply linear programming and heuristics. In case of integer demand specific enumeration techniques or sometimes mixed integer programming are used, working fine for smaller models. Continuous approximations can also provide lower bounds.

2.3. List of used symbols

The following provides the list of the notations that we will use throughout the paper. The subscript j refers to inventory item j of the multi-product portfolio.

Cost notations:

- c_j purchase cost, variable cost to place an order (*moneys/quantity-unit*)
- c_{oj} overage cost for remaining inventory at period end in newsvendor problem (*moneys/quantity-unit*)
- c_{uj} underage cost for unsatisfied period demand in newsvendor problem (*moneys/quantity-unit*)
- h_j holding cost, cost to hold one unit in inventory for one unit of time (*moneys*/[*quantity-unit*∗*time-unit*])