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# Non-cooperative consignment stock strategies for management in supply chain

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ABSTRACT

This paper analyzes the coordination and competition issue in a two-level supply chain, having one vendor (or manufacturer) and one buyer (or retailer). A continuous deterministic model is presented. To satisfy the buyer's demands, the product is delivered in discrete batches from the vendor's stock to the buyer's stock and all shipments are realized instantaneously. We describe inventory patterns and the cost structure of production–distribution cycles (PDC) under generalized consignment stock (CS) policies. For the joint optimization case, the average total cost of production, shipment and stock-holding is minimized. Optimal solution techniques are presented and illustrated with numerical examples.

In a competitive situation, the objective is to determine schedules, which minimize the individual average total costs in the PDC obtainable by individual decisions. This paper presents a non-cooperative two-person constrained game with agents (a vendor and a buyer) choosing the number and sizes of deliveries. Generalized CS-policies are considered as feasible individual strategies in the game. We consider the class of non-cooperative sub-games, indexed by two integer parameters connected with CS policies. It is proven that there exists a unique Nash equilibrium strategy in each of the considered sub-game.



## 1. Introduction

Supply chain management has recently received a great attention in economics. In general, a supply chain is composed of independent partners with individual costs. When applied to productive environments, it allows (for isolated situations) the vendor to calculate the Economic Production Quantity (EPQ), although it might be significantly different from the buyer's EOQ. One of the major task of management is to coordinate the processes of the supply chain in such a way that the lowest system-wide cost is gained.

The idea of joint optimization for vendor and buyer was initiated by Goyal (1976), Banerjee (1986) and Lu (1995). Several authors (see the literature review by Sarmah et al. (2006)) incorporated policies in which the sizes of successive shipments (from the vendor to the buyer within a production cycle) either are equal in size or increases by a factor equal to the ratio of production rate to the demand rate. Hill (1999) combining these two ideas shows that in an optimal cycle the total production is transferred in deliveries of initially increasing and then equal size. However, there is an additional set of problems involved in implementing policies (strategies) with respect to whether and how the agents participate in the delivery-transportation costs.

Some researchers suggest quantitative models to describe the motivation and negotiating tools for providing joint operating policies. There is a lot of research done dealing with problems of coordinating a distribution system under vendor-managed inventory and consignment arrangements (Gümüs et al., 2008; Chen et al., 2010; Ru and Wang, 2010). An analytic formulation of consignment stock (CS) is discussed in Braglia and Zavanella (2003) and Zanoni and Grubbström (2004). In the case of CS-policies there are three decisions to take: about the delivered amount, about the number of deliveries making up the production batch, and how many deliveries should be delayed once this amount is available for shipment. For the case of multiple buyers, see Zavanella and Zanoni (2009) where an analytic formulation of consignment stock policies is given as well as the exhaustive references to this subject.

Generalized consignment stock $(k,n)$-policy (say CS$(k,n)$-policy) requires that:

- initial $k \geq 0$ deliveries are identical in sizes (say, of the size $q^v$),
- preceded $n \geq 0$ deliveries are also equal in sizes, say $q^b$.

Zanoni and Grubbström (2004) consider the case where $q^v = q^b$. Some relations for individual costs of inventories (in the

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case when the agents can make decisions according to their own preferences) were analyzed by Gümüs et al. (2008).

In this paper we assume that the delayed deliveries are dispatched to the buyer's stock as late as it is possible instead of previously assumed "as soon as possible". For the central coordination case, such policy has lower cost only if the unit stockholding cost increase as stock moves down the supply chain. An analytic formulation of the modified CS policy is presented and an implicit solution is given.

The second part of this paper presents a game-theoretic approach for the case with non-equal sizes of deliveries. A model is considered, where agents minimize their individual costs under the assumption that only the division of shipment costs is coordinated centrally or negotiated. The CS($k$,$n$)-policy can be viewed, from competition perspective, as a pair of non-cooperative vendor–buyer strategies, analogously as Hill's policies in Bylka (2009).

The paper is organized as follows. In Section 2, we develop the model describing inventory patterns under ($k$,$n$)-consignment stock policies. The costs of optimal policies in the considered classes CS($k$,$n$)-policies are given in Sections 3 and 4. Formulas for Nash equilibrium strategies in the restricted non-cooperative games are presented in Section 5.

## 2. Modeling vendor–buyer relationships under CS($k$,$n$)-policies

We consider a continuous deterministic model of a production–distribution system for a single product. A vendor produces a product on a single machine and supplies it to the buyer. Buyer's demand is represented as a continuous function of the time. We denote by $P$ the vendor's production rate, by $D$ the buyer's demand rate and the ratio $\lambda = P/D$.

We examine the situation, where a vendor produces a product in a batch production environment and supplies it to the buyer under deterministic conditions (see Goyal, 1976). A schedule is determined by a sequence of production–distribution cycles, each of them determined by the following quantities:

$Q$=the size of production batch;
$m>0$=the number of shipments per production–distribution cycle;
$(q_j,t_j)$=quantity and the moment of $j$-th shipment, $j=1,\ldots,m$.

The problem, in question, can be characterized as follows.

A1. Constant production rate is sufficient to meet buyer's demand ($\lambda>1$) and buyer's demand must be satisfied.
A2. The final product is distributed by shipping it in discrete lots from the vendor's stock to buyer's stock (realized instantaneously).
A3. The vendor's stock becomes empty just past each of the first set of (uniform size) deliveries, while the remaining (uniform size) deliveries are sent as late as it is possible. *This assumption will be commented below.*

In the considered model, the vendor and the buyer cost parameters are the following:

$A$=fixed production set up cost;
$h_i$=unit stock holding cost for the vendor, $i$=0, and for the buyer $i$=1;
$A_i$=delivery (transportation and replenishment) cost per shipment, $i$=0,1.

The objective is to determine a production and shipment schedule which minimizes the average (joint or buyer–vendor individual) total cost of production, shipment and stockholding. A policy ought to define a mode of decision making which determined a schedule. Policy can be given as a pair of strategies (vendor's and buyer's decision functions).

### 2.1. The production–distribution cycle

In each production–distribution cycle (PDC) the production starts at a moment, say 0, when the buyer have some initial inventories $q_0$. A schedule

$$\tilde{q} = [(q_1,t_1),\ldots,(q_m,t_m)] \quad \text{where} \quad \sum_{j=0}^{i-1}\frac{q_j}{D} \ge t_i, \quad i=1,\ldots m$$

determines the number, quantities and timing of successive shipments to the buyer. The production is stopped at the moment $t^*$ and it starts again in the next cycle at $T$ (the end moment of the PDC) with the buyer's final inventories $q_0$. The idle time is equal to $T-t^*$. We have

$$Q=\sum_{j=1}^{m} q_j, \quad T=\frac{Q}{D} \quad \text{and} \quad t^*=\frac{Q}{P}\left(=\frac{T}{\lambda}\right). \tag{1}$$

Let us denote (with the notation $i$=0, for the vendor and $i$=1 for the buyer):

$I_i(t)$=the inventory position at $t$ just before the possible replenishment;

$I_1(0)=q_0$=the buyer's initial inventory position.

Therefore, for a moment $t\in[0,T]$ we have the following inventory positions:

$$I_0(t)=P\min\{t,t^*\}-\sum\{q_j \mid t_j<t\},\; I_0(t^+)=P\min\{t,t^*\}-\sum\{q_j \mid t_j\le t\},$$

$$I_1(t)=I_1(0)-Dt+\sum\{q_j \mid t_j<t\}, \quad \text{with } t_j\le t \text{ for } I_1(t^+) \tag{2}$$

and the individual total stock holding quantities in the cycle

$$\bar{I}_0=\int_0^T I_0(t)\,dt \quad \text{and} \quad \bar{I}_1=\int_0^T I_1(t)\,dt.$$

Let us note that even each of $\bar{I}_0$ and $\bar{I}_1$ depends on the structure of the schedule $\tilde{q}$, the common total stock holding quantity $\bar{I}=\bar{I}_0+\bar{I}_1$ depends only on $q_0$. The main problem consists of the allocation of total stored quantity $\bar{I}$ in vendor's and buyer's stocks.

The above assumptions A1–A3 have an inspiration in practice and they are natural assumptions in joint buyer and vendor coordination models. Also, one of the following properties was postulated in the literature:

3a. The buyer receives all deliveries just to run out of the stock.
3b. The vendor's stock becomes empty just past each delivery.
3c. All shipments are identical, i.e. $q_1=\cdots=q_m$.
3d. The vendor's inventory is zero just past some initial replenishment and the successive (uniform in size) deliveries are sent *as soon as it is possible*.
3e. The vendor's inventory is zero just past some initial replenishment and the successive (uniform in size) deliveries are sent *as late as it is possible*.

According to some important (for the theory and the practice) classes of policies with respect to the above properties, we say that a policy is:

- a Goyal's policy if it satisfies postulates 3a and 3b (Goyal, 1976);
- an equal size delivery policy if 3c and 3a (Lu, 1995);
- a Hill's policy if 3a and 3e (Hill, 1999)—it implies that the initial deliveries increase in sizes with the rate $\lambda$;