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A new class of stock-level dependent ordering policies for perishables with a short maximum shelf life

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ABSTRACT

Many perishables such as fresh food and blood platelet concentrates are characterized by a short maximum shelf life. As demand is often highly uncertain the outdating and shortages figures can be very high, especially when frequent replenishment is not possible or inefficient due to fixed ordering cost. We present a new class of stock-level dependent ordering policies: the (s,S,q,Q) policy, which is a periodic review (s,S) policy with the order quantity restricted by a minimum (q) and maximum (Q). Optimal (weekday dependent) parameter values are derived by dynamic programming and simulation. The (s,S,q,Q) policy performs nearly optimal and improves the (s,S) policies in many cases by 4–25%.

1. Introduction

1.1. Ordering problem and assumptions

We focus on inventory systems of perishables products for which balancing outdating and shortages is a challenging problem. Typical products that we have in mind are fresh produce, dairy products, and blood products with a short maximum shelf life of $m=3$–7 periods. Inventories of such products are characterized by significant outdating and shortages when demand is highly uncertain and replenishments do not happen every period. In Haijema et al. (2007) a stochastic dynamic programming–simulation approach is presented for the periodic review of inventories of blood platelet concentrates. We copy the main assumptions of that paper, i.e. the demand is stochastic and periodic, the product lead time is 1 period, the periodic placement of an order is forbidden in some periods (e.g. during weekends). In this paper we are generalizing this model by including *fixed ordering costs* that are involved in placing an order or setting up a production run. In Haijema et al. (2007) these fixed costs could be neglected for the specific case of a Dutch central blood bank, but fixed costs should be accounted for in many other settings (e.g. hospitals and blood banks with much smaller inventories that are not replenished daily). As a result, the optimal ordering policy is more complex and we derive a new class of ordering policies and a new procedure to determine nearly optimal parameter values for these policies.

1.2. Approach and main results

We will formulate the ordering problem as a periodic Markov decision problem (MDP). To ease the discussion we set the review period to 1 day and assume that the problem is (1) stationary across weeks (any week is stochastically the same as any other week) and (2) non-stationary within a week (i.e some problem parameters are different for the different weekdays).

For a variety of cases we derive numerically an optimal *stock-age dependent* ordering policy by stochastic dynamic programming (SDP). Via simulation the structure of the optimal policy is investigated, and *stock-level dependent* policies that fit well are extracted. Stock-level dependent policies are preferred for practical use as these are easier to understand and to apply by inventory managers. The optimal policy can be approximated by a 'periodic' (s,S) policy that has weekday (d) dependent parameter values (s_d,S_d).

The structure of the optimal policy suggests to restrict the order quantity by a minimum and a maximum, q_d respectively Q_d, on weekday d. We call our new policy the periodic (s,S,q,Q) policy or the (s_d,S_d,q_d,Q_d) policy. This class of stock-level dependent ordering policy appears to be new and performs in many cases close to optimal and much better than the (s_d,S_d) policy. The SDP–simulation approach provides not only a benchmark by computing an optimal stock-age dependent policy, but it also derives near optimal parameter values for both the (s_d,S_d) policy and the (s_d,S_d,q_d,Q_d) policy.

1.3. Outline

In the next section (Section 2) we discuss relevant literature. In Section 3 we formulate the ordering problem as an MDP.

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For a numerical base case, we compute and simulate in Section 4 an optimal stock-age dependent ordering policy. Next, we approximate it by stock-level dependent policies (see Section 5). In Section 6 we compare for a variety of cases the optimal policy and the approximate ordering policies. In Section 7, we close the paper and conclude when to consider the new class of ordering policies.

2. Literature review

Most of the research of ordering polices for perishable is dedicated to *stock-level dependent* policies, like order-up-to S and (s,S) policies. These policies do not acknowledge the perishable nature of the product and are therefore generally suboptimal, see Nahmias (1975b) and Fries (1975). For an overview of the early work we refer to Nahmias's (1982) overview. Recently an accurate overview of about 150 papers is provided by Karaesmen et al. (2011). None of these papers present a periodic review model for a perishable product with a fixed shelf life, a fixed lead time and fixed ordering costs.

Nahmias (1975a) notes that the optimal order quantity for a perishable product is often lower than the quantity for a non-perishable product (NP) set by an order-up-to policy with fixed order-up-to level S^{NP}. Therefore he develops and simulates two modifications of the order-up-to S^{NP} policy. The first modification is to order linearly less than in the non-perishable case: that is order $\beta(S^{\mathrm{NP}}-x)$ products, where x is the total stock level and $(0<\beta<1)$. The second modified policy includes an upper bound Q on the order quantity: that is order $\min\{Q,S^{\mathrm{NP}}-x\}$. It appears that both modified policies perform worse than an order-up-to S^{*} policy with $S^{*}\leq S^{\mathrm{NP}}$ being the optimal order-up-to level for the perishable case. Nahmias neither optimize over S and Q simultaneously, nor did he study in the impact of fixed ordering costs and the inclusions of a lower bound on order quantity as we do for the (s,S,q,Q) policy. We will observe that including bounds q and Q does make sense.

In a lost sales model for non-perishables with a positive product lead time it is shown, under a common linear costs structure that the optimal policy is restricted by a maximum order quantity Q (see Hill and Johansen, 2006). This maximum limits the order quantity when the stock level is low and thus anticipates that part of the demand may be lost during the lead time.

A lower bound on the order quantity is studied only in a few papers on non-perishables. In all papers, this minimum order quantity is a fixed problem parameter that is not to be optimized.

Finding a cost-optimal ordering policy for perishables with a fixed maximum shelf life is complicated because as such an optimal policy is typically *stock-age dependent* resulting in a multi-dimensional state space. In Haijema et al. (2007) an aggregation approach is used to overcome the computational complexities and simple policies are derived to be used in blood management practice. In that paper any fixed set-up costs in producing blood platelet concentrates for a central blood bank could be neglected as production happens every weekday. In that setting the optimal policy can be closely approximated by an order-up-to S policy. In Haijema et al. (2009) the same problem but with non-stationary production breaks is solved.

2.1. Contribution

We derive a new *class* of stock-level dependent ordering policies that includes and improves existing ones. The numerical results contradict earlier observations of Nahmias (1975a): adding bounds to a base stock policy may greatly improve the ordering policy for perishables! We focus on inventories problems in which outdating and shortages are significant even under an optimal stock-age dependent ordering policy, e.g. because of non-negligible fixed ordering costs. Therefore we present a new class of policies and a new procedure to find (nearly) optimal parameter values.

3. Optimal ordering: a Markovian decision problem (MDP)

An optimal ordering policy can be computed by formulating and solving the underlying Markov decision problem (MDP). For the details we refer to Haijema et al. (2007). The state of the MDP is $(d,\mathbf{x})$ where $d\in\{1,\ldots,7\}=\{\text{Monday},\ldots,\text{Sunday}\}$ and $\mathbf{x}=(x_1,\ldots,x_m)\in\mathcal{X}(d)$ with element x_r equal to the number of products in stock at the start of the day that have a residual shelf life of r days. If not issued x_1 products expire by the end of the day. After observing the state one decides to order $a\in\mathcal{A}(d,\mathbf{x})=$ products at the start of day d.

We consider two types of demand: type 1 customers get the youngest available products (LIFO), type 2 customers get the oldest products first (FIFO). The stock transition from one state $\mathbf{x}$ to a next state $\mathbf{y}$ is denoted by the function $\mathbf{y}(\mathbf{x},j_1,j_2,I,a)$, where j_1 and j_2 are the quantities demanded by the two types of demand and I is the composite issuing policy, e.g. (LIFO, FIFO). In this transition shortages occur when $j_1+j_2>x=\sum_{r=1}^{m}x_r$. Any product left in stock will age by 1 day at the end of the day. Then outdated products are removed from stock and new products arrive prior to the next stock inspection: $y_m=a$. In order to track outdating, let $j_{1,1}^{I}$ and $j_{2,1}^{I}$ denote the number of products with only 1 day of residual shelf life that is taken from stock to meet the type 1 respectively type 2 demand. The number of products that outdate is thus $(x_1-j_{1,1}^{I}-j_{2,1}^{I})^{+}$, with $z^{+}=\max\{0,z\}$.

The expected immediate costs to incur in state $(d,\mathbf{x})$ when a products are ordered are

$$\mathbb{E}C(d,\mathbf{x},a,I)=\sum_{j_1,j_2}p_d(j_1,j_2)\cdot\begin{cases}c^{\mathrm{F}}\cdot\delta(a>0) & \text{fixed order costs,}\\ +c^{\mathrm{O}}\cdot(x_1-j_{1,1}^{I}-j_{2,1}^{I})^{+} & \text{outdating costs,}\\ +c^{\mathrm{S}}\cdot(j_1+j_2-x)^{+} & \text{shortage costs,}\end{cases}\tag{1}$$

where $p_d(j_1,j_2)$ is the convolution of two (discrete) demand distributions and $\delta(a>0)=1$ when $a>0$ and 0 otherwise. For each order that is placed fixed ordering costs c^{F} are accounted that relate to the effort involved with a replenishment (e.g. the transhipment and handling costs or the costs of setting up a production run). The unit outdating costs c^{O} are the effective costs of disposing a product, which include the purchase or production costs of the outdated product. The unit shortage costs c^{S} impose a penalty on being out-of-stock and include, if applicable, the unit costs related to a lateral transhipment. Note that we have kept the cost structure simple; one could add holding costs. In case age preferences are formulated one may add penalties on issuing older products.

The objective is to find an ordering policy that minimizes the expected average costs per period, which we can compute by stochastic dynamic programming (see Puterman, 1994):

$$V_n(d,\mathbf{x})=\min_{a\in\mathcal{A}(d,\mathbf{x},I)}\left(\mathbb{E}C(d,\mathbf{x},a).+\sum_{j_1,j_2}p_d(j_1,j_2)V_{n-1}(d+1,\mathbf{y}(\mathbf{x},j_1,j_2,I,a))\right).\tag{2}$$

When the number of states is very large the discrete state and action spaces could be aggregated to speed up the computations (see Haijema et al., 2007).