Int. J. Production Economics 143 (2013) 440–448

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Determination of safety stocks in a lost sales inventory system with periodic review, positive lead-time, lot-sizing and a target fill rate

Karel H. van Donselaar*, Rob A.C.M. Broekmeulen

Eindhoven University of Technology, Department of Industrial Engineering & Innovation Sciences, Paviljoen Room F2, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

ARTICLE INFO

Article history:
Received 14 September 2010
Accepted 17 May 2011
Available online 26 May 2011

Keywords:
Inventory
Lost sales
Safety stock
Fill rate
Approximation
Service level
Dual sourcing

ABSTRACT

An approximation for the fill rate, i.e. the percentage of demand being delivered from inventory on hand immediately, is derived for items in a periodic review inventory control system with lost sales. We assume demand is stochastic and discrete, lead-times are positive and replenishments are made in multiples of a given fixed case pack size. Most literature on inventory control systems assumes that unmet demand is backordered. The major reason for this is that the analysis of a general lost sales inventory system is known to be hard. To find an approximation for the fill rate, given a safety stock, we start with existing analytical approximations. By applying linear regression, we slightly modify these existing approximations. The new approximation is tested for a wide set of parameters and performs very well: the average approximation error for the fill rate is only 0.0028 and the standard deviation of the approximation error is 0.0045. Since the approximations are very fast, this result enables inventory controllers dealing with a lost sales inventory system to set safety stocks in accordance with the target service level set by their management in an effective way. The results of our study also show that the assumption that the lost sales system can simply be approximated by a backordering system if the target fill rate is at least 95%, may lead to serious approximation errors. These errors are particularly large when the lead-time is large or demand uncertainty is low and when on average there is at least one replenishment order outstanding.

1. Introduction

Many inventory systems in practice are confronted with lost sales if demand cannot be delivered instantaneously from inventory on hand. In a worldwide survey on out-of-stocks in the fast moving consumer goods retail sector for example, Gruen et al. (2002) show that only 15% of all customers delay the purchase if they are faced with a stock-out for their preferred product in a particular store. The other 85% of the customers decide to buy another product (substitution), buy the product in another store or not buy the product at all. In all these cases demand for the preferred product–store combination is lost.

While lost sales systems are very relevant in practice, most scientific papers on stochastic inventory models assume backordering. The reason for the limited attention for lost sales systems in the scientific literature is the fact that discrete-time inventory models with stochastic demands, a constant lead-time and lost sales are notoriously difficult (Zipkin, 2008). The first structural results were derived by Karlin and Scarf (1958) and Morton (1969). In contrast to the case with backordering, where the optimal reorder quantity can be derived from a single number (the sum of the inventory on hand and on order), in case of lost sales the optimal reorder quantity is a function of the inventory on hand as well the timing and quantity of all outstanding orders. As a result, the state space increases rapidly as the lead-time increases.

For an extensive and recent review of the literature on lost sales systems we refer to Bijvank and Vis (in press). They classify the literature in four categories. They subsequently discuss continuous review systems with fixed or variable order sizes (referred to as (s,Q) and (s,S) policies) and periodic review systems without and with fixed order costs. Their literature review confirms that there are only a limited number of papers dealing with lost sales systems and the vast majority of these papers make simplifying assumptions to make them analytically tractable. Out of the 61 papers they identified, 18 papers study continuous review systems with a fixed order size and the majority of these papers (13 papers) assume at most 1 (or 2) orders are outstanding, while four other papers assume lead-time and/or demand have a specific probability distribution function (e.g. Poisson). Likewise, out of the eleven papers on continuous review systems with variable order sizes, i.e. (s,S)-order-up-to policies, nine papers assume $s=S-1$. The majority of the 32

* Corresponding author. Tel.: +31 40 2472869.
E-mail addresses: k.h.v.donselaar@tue.nl (K.H. van Donselaar), r.a.c.m.broekmeulen@tue.nl (R.A.C.M. Broekmeulen).

0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2011.05.020

papers on periodic review systems assume there are no fixed order costs (24 papers) and out of the remaining 8 papers only one published paper, by Johansen and Hill (2000), deals with fixed order sizes. Their paper assumes that at most one order is outstanding, which limits its applicability.

Another observation from this literature review is the fact that in the majority of the 61 papers on lost sales systems the objective is to minimize costs, while only eight published papers are based on a minimal service restriction. These eight papers are: Hadley and Whitin (1963), Mohebbi and Posner (1998), Hill (1992, 1994), Aardal et al. (1989), Tijms and Groenevelt (1984), Van Donselaar et al. (1996) and Kapalka et al. (1999). Very recently, an approximation for the fill rate service criterion in a lost sales system with a (R,s,nQ)-policy was derived, which was developed and tested for an environment with perishable items having relatively small lead-times, small case pack sizes and low demand uncertainty (Van Donselaar and Broekmeulen, 2011). Bijvank (2009, pp. 57–58) claims that adding a minimal service restriction to an inventory model with lost sales makes the model more realistic to represent a retail environment, but also makes the analysis and computations more difficult. He then compares the (R,s,nQ)-policy with the optimal policy in a periodic review setting with a service level constraint for a wide set of inventory systems and concludes that the (R,s,nQ)-policy is close to optimal (around 0.3% deviation), but that there is a need for a good approximation procedure to set the control parameters. Recently Zipkin (2008) evaluated base stock policies in lost sales systems and identified specific areas where they are more than 5% from optimal: if the lot-size is very small (the ratio between case pack size and average demand per review period was equal to 0.2 in his experiments), the penalty costs for lost sales are low and the lead-time is large. If these three conditions are met simultaneously, it is suggested to apply one of the other policies investigated by Zipkin.

Our paper is the first paper studying for a wide set of product- and demand-parameters the determination of the reorder level in a lost sales system with a (R,s,nQ)-policy in a periodic review setting with a service level constraint. We derive and test an approximation for the fill rate as a function of the safety stock and a fixed case pack size (often determined by an external supplier). Based on the literature we first identify two constructs of variables having a large impact on the performance of lost sales systems. Next we study the behavior of three currently existing approximations for the fill rate in lost sales systems as a function of these constructs of variables. Finally we use linear regression and the constructs of variables to improve the current approximations. Ehrhardt (1979), Ehrhardt and Mosier (1984) and Schneider and Ringuest (1990) also applied linear regression to set reorder levels, but did this for periodic review inventory systems with backordering and (s,S)-policies. Berling and Marklund (2006) also used linear regression in the context of inventory control systems. They used this technique to get an approximation for the induced backorder costs in a one-warehouse multiple-retailer system.

We compare the performance of the new approximation with the current approximations and show that the new approximation performs very well: the average and standard deviation of the approximation error is below 0.5%. Since the approximations are very fast, this result enables inventory controllers dealing with a lost sales inventory system to set safety stocks in accordance with the target service level set by their management in an effective way. The approximations perform substantially better than traditional approximations, especially in environments where either the lead-time is large or demand uncertainty is low and when on average there is at least one replenishment order outstanding.

The structure of this paper is as follows. In Section 2 the model assumptions and notation are introduced. Section 3 introduces and evaluates three currently existing approximations. The new approximation is developed and tested in Section 4. Section 5 provides simple guidelines for inventory managers on the potential performance improvement when the results of this paper will be applied in practice. Section 6 gives the conclusions of this paper and suggestions for future research.

2. Model assumptions and notation

We study a single echelon inventory system having a positive lead-time L, a review period R, a fixed case pack size Q and stochastic discrete demand per period with mean μ and variance σ^2. Demand which is not met from stock is lost.

In the model, the sequence of events during a period is as follows: first demand is subtracted from the inventory during the period, performance measures such as the service level are calculated, goods arrive, and finally the orders are placed.

The reorder policy is a simple (R,s,nQ)-policy; if at a review moment the inventory position drops below the reorder level s, an integer multiple of case packs each with fixed size Q units are ordered in order to raise the inventory position back to or above s. The reorder level s is equal to $s=(L+R)\mu+ss$, with ss denoting the safety stock. The service level of the inventory system is measured via the service level P_2, also known as the fill rate. The fill rate is defined as the percentage of demand which can be satisfied from the inventory on hand immediately.

To evaluate the quality of the approximations to be derived in this paper, we simulated a very large number of inventory systems by changing all six system-parameters in a systematic way: the average demand, the variance to mean ratio for the demand, the case pack size, the lead-time, the review period and the safety stock. Table 1 shows which values were attributed to these parameters. These values are in line with the values in the design of experiments by Bijvank (2009). For example, Bijvank used the values 2, 5 and 10 for the average demand and varied the variance to mean ratio from 1 to 3.5 (depending on the demand distribution used). We use the same average demand, but also include lower and higher variance to mean ratios (in line with Johansen and Hill (2000), who also included experiments with low demand uncertainty). For the discrete demand distribution we used the fitting procedure of Adan et al. (1995), which includes the binomial, negative binomial, Poisson and geometric distribution. Bijvank assumed a (compound) Poisson or negative binomial distribution. To further enlarge the design of experiments, also inventory systems with very small case pack sizes (including the lot-for-lot policy and fixed case pack sizes which are equal to the average demand per period) and large lead-times were added, in line with Nahmias (1979) who varied the lead-time from 5 to 10 and 20 periods.

For every combination of parameter values we varied the safety stock, starting at value zero and increasing it in steps of 1 unit until a fill rate of 99% was passed. This implies that our experimental design also includes experiments with a low fill rate (51.3%) as well as experiments with a very high fill rate (99.95%).

Table 1
Input parameters for the simulation experiment.

Input parameter	Levels
Mean demand per period μ	{2, 5, 10}
Variance-to-mean ratio σ^2/μ	{0.1, 1, 3, 10}
Case pack size Q	{1μ, 2μ, 5μ, 10μ}
Lead-time L	{1, 2, 5, 10, 20}
Review period R	{1, 2}