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Initiation of an inventory control system when the demand starts at a given time

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ABSTRACT

This paper considers a single-echelon, continuous review, inventory system with a warehouse facing compound Poisson customer demand. The replenishment lead-time is constant. Demand that cannot be met directly is backordered. There are standard linear holding and backorder costs but no set-up or ordering cost. It is assumed that the demand process starts at a certain given time. Consequently, before the demand starts, the lead-time demand is lower than in steady state. This affects the optimal ordering policy. We derive the optimal ordering policy under these assumptions.

1. Introduction

A standard assumption in stochastic inventory models is that we have reached steady state. The optimal policy is therefore only directly applicable under this assumption. However, there is normally also an initiation problem when starting to sell a new item. It may be that the company announces officially that it starts to sell a new item at a certain date. Then there is no demand before that date.

In this paper we consider such a situation. The stochastic demand in the form of a compound Poisson process starts at a certain time denoted time 0. Otherwise all assumptions are standard. There are holding and backorder costs per unit and unit time. There is no set-up or ordering cost so there is no advantage to order in batches. The replenishment lead-time is constant.

Several previous papers have dealt with related initiation problems in connection with inventory control. One such situation, considered in several papers, is when demand cannot be satisfied until a batch is delivered, there is no initial stock, and the production rate is finite. It is then, in general, optimal to use smaller initial batch quantities so that demand can be satisfied earlier. Examples of models dealing with this aspect are Axsäter (1988), Ding and Grubbström (1991), and Grubbström and Ding (1993). Axsäter (in press) considers a related situation where the forecasts are improving. It turns out that this will also affect the initial batch quantities. Other reasons to use different initial batch quantities can be learning and forgetting effects, which change the production rate. See e.g., Elmaghraby (1990) and Klastorin and Moinzadeh (1989).

This paper is organized as follows. Section 2 describes the considered problem in detail. We derive the optimal policy in Section 3. Finally, we give a few concluding remarks in Section 4.

2. Problem formulation

A single warehouse facing discrete compound Poisson customer demand is considered. Demands that are not met directly are backordered. We consider standard holding and backorder costs. There are no ordering costs. The lead-time for replenishments is constant. Furthermore, we assume continuous review.

Under the considered assumptions, it is well known that the optimal control policy in steady state is an $(S-1, S)$ policy, or equivalently an S policy, i.e., when the inventory position (stock on hand, plus outstanding orders, and minus backorders) declines below the optimal order-up-to level S, an order is triggered to bring the inventory position back to S. Such policies are common in practice especially for relatively expensive spare parts with low demand.

However, we do not consider a system in steady state. Instead, it is assumed that the demand process starts at time 0, i.e., before this time there is no demand. We can order at any time before or after time 0, though, and the replenishment lead-time is constant and the same for all orders.

Let us introduce the following basic notation:

L	lead-time for replenishments,
S	order-up-to inventory position,
$D(t)$	stochastic demand during time t,
λ	customer arrival intensity,
f_j	probability for demand quantity j, $f_j=0$ for $j<1$,
f_j^n	probability that the total number of units demanded by n customers is j, i.e., the n-fold convolution of f_j,
μ	$\sum_{j=1}^{\infty} jf_j=$average size of a customer demand,

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h nonnegative holding cost per unit per unit time,
b nonnegative backorder cost per unit per unit time,
IP inventory position,
IL inventory level,
$\text{Po}(\lambda,k)=\sum_{j=0}^{k}\frac{e^{-\lambda}\lambda^j}{j!}$ cumulative Poisson distribution.

3. The optimal policy

3.1. Optimal steady state policy

Consider first a steady state situation. It is well known how to determine the expected costs when applying an $(S-1, S)$ policy. (See e.g., Axsäter, 2006 for more details.) Consider an arbitrary time t and also the time $t+L$. Orders that are triggered in the interval $(t, t+L]$ have not reached the inventory at time $t+L$ because of the lead-time, but everything that was already on order at time t has reached the inventory. Consequently we have

$$IL(t+L)=IP(t)-D(t,t+L) \tag{1}$$

where $D(t, t+L)$, or simpler $D(L)$, is the stochastic lead-time demand.

Assume that the inventory position is kept at S all the time and let the corresponding expected holding and backorder costs per unit of time be $C(S, L)$. Using (1) we get

$$C(S,L)=hE(IL^+)+bE(IL^-)=(h+b)E(IL^+)-bE(IL)$$
$$=(h+b)e^{-\lambda L}\sum_{j=0}^{S-1}(S-j)\sum_{n=0}^{j}\frac{(\lambda L)^n}{n!}f_j^n-b(S-\lambda L\mu) \tag{2}$$

where we use the notation: $x^+=\max(x, 0)$ and $x^-=\max(-x, 0)$. Note that $x=x^+-x^-$. In (2) n is the number of customers during the lead-time, which has a Poisson distribution. Furthermore j is the corresponding demand. Recall that f_j^n is the probability that n customers result in the total demand j. The inventory level at time $t+L$ is $S-j$.

$C(S, L)$ is convex in S, and $S=0$ is a lower bound for the optimal order-up-to level. (A lower S will just give higher backorder costs but no reduction of the holding costs.) We can therefore optimize S by starting with the lower bound $S=0$ and then increase S by one unit at a time until we find a local optimum, which is then also a global optimum. We denote the optimal policy $S^*(L)$ and the corresponding optimal cost $C^*(L)$. It is also well known that $S^*(L)$ is nondecreasing in L. (This follows e.g., from (5.61) in Axsäter (2006), p. 103.)

3.2. Optimal initiation policy

Let us now turn to the case when the demand starts at time 0. It is obvious that we do not want any deliveries before time 0. This means that we should have no orders before time $-L$. We can get deliveries at any time $t\geq 0$ by ordering at time $t-L$. Let us now first state the following simple proposition.

Proposition 1. From time 0 it is optimal to apply an $S^*(L)$ policy.

To see that Proposition 1 is true we just note that (1) and (2) are valid for $t\geq 0$ and that we get the optimal policy in the same way as we obtained the optimal steady state policy in Section 3.1.

Next we consider the policy in the remaining interval $[-L, 0)$ and we have

Proposition 2. In the interval $[-L, 0)$ it is optimal to apply a time variable $(S-1, S)$ policy. At time t the optimal order-up-to-level is $S_t^*=S^*(t+L)$. (Note that when t is increasing from $-L$ to 0 the considered interval length $t+L$ is increasing from 0 to L.)

Proof. Consider an ordering policy in $[-L, 0)$ and let IP_t be the accumulated orders at time t, i.e., the orders in the interval $[-L, t]$. Note that IP_t must be nondecreasing. Recall that there is no demand in $[-L, 0)$. All that is included in IP_t has reached the stock at time $t+L$, while later orders have not. Consequently we get the inventory level at time $t+L$ as

$$IL(t+L)=IP_t-D(0,t+L)=IP_t-D(t+L). \tag{3}$$

From the derivation of the optimal policy in steady state in Section 3.1, it is obvious that we minimize the expected costs at $t+L$ if $IP_t=S^*(t+L)$. This is then also the optimal policy, because $S^*(t+L)$ is nondecreasing so we can follow this inventory position for all $t<0$ by applying the order-up-to-level $S_t^*=S^*(t+L)$. This proves the proposition. □

3.3. Numerical determination of the decision rule

To be able to apply the optimal policy we need to determine $S_t^*=S^*(t+L)$ for different values of t. For $t=-L$ we have $S_{-L}^*=S^*(0)=0$ because the cost rate is then 0 at time 0. Just before $t=0$ we have $S_0^*=S^*(L)$. As we increase t from $-L$ to 0, S_t^* will increase stepwise, one unit at a time, from $S^*(0)=0$ to $S^*(L)$. Clearly, the switch from S to $S+1$ must occur when $C(S, t+L)=C(S+1, t+L)$. Note that a switch must be for one unit at a time. To see this, assume there is a switch from S to $S+k$ and that $k>1$. We must then have $C(S, t+L)=C(S+k, t+L)$, and both S and $S+k$ are optimal. But due to the convexity, intermediate values e.g., $S+1$ will give even lower costs, which is a contradiction.

It is easy to determine the optimal switching points by a bisection search. Let t^k be the time when it is optimal to switch to order-up-to level k. Recall that the largest value of k that we need to consider is $S^*(L)$. Assume that t^{k-1} is known. (Recall that $t^0=-L$.) Clearly, t^{k-1} is lower bound for t^k, i.e., $\underline{t^k}=t^{k-1}$. As an upper bound we can use $\overline{t^k}=0$. We can get a new value of the switching time as $t=(\underline{t^k}+\overline{t^k})/2$. Clearly, if $C(k-1, t+L)<C(k, t+L)$ we know that t can serve as an improved lower bound, and otherwise as an improved upper bound. We continue like this until the gap between the bounds is sufficiently small.

3.4. Example

To illustrate the determination of the optimal policy we consider an example with pure Poisson demand, i.e., $f_1=1$. Furthermore, the intensity of the customer arrivals is $\lambda=1$. The lead-time $L=2$, the holding cost per unit and unit time is $h=1$, and the backorder cost per unit and unit time $b=10$. Using (2) we determine the optimal steady state order-up-to level as $S^*(2)=4$. Next we determine the switch points from 0 to 1, 1 to 2, 2 to 3, and 3 to 4. We get the times -1.905, -1.498, -0.944, and -0.315 respectively. The optimal order-up-to policy is illustrated in Fig. 1.

3.5. Cost evaluation

We know that the optimal order-up-to inventory position is piecewise constant and increasing by one unit at a time in the interval $[-L, 0)$. The policy in this interval will determine the transient costs in the interval $[0, L)$. We shall determine the expected costs in this interval. After this interval we have the optimal steady state costs that we do not need to consider again, so we limit our attention to the costs in the interval $[0, L)$. Let the optimal inventory position be S in the interval $[\alpha, \beta]$, where $-L\leq\alpha\leq\beta\leq 0$. The corresponding expected costs in $(\alpha+L, \beta+L)$ are denoted by $c(\alpha, \beta)$. Consider a time t in the considered interval. The corresponding lead-time demand is the compound Poisson demand during the time $t+L$. Recall that the demand starts at time 0.