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# Worst case analysis of flow shop scheduling problems with a time-dependent learning effect

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ABSTRACT

In this paper we consider flow shop scheduling problems with a time-dependent learning effect. The time-dependent learning effect of a job on a machine is assumed to be a function of the total normal processing time of the jobs scheduled in front of the job on the machine. The objective is to minimize one of the five regular performance criteria namely, the total completion time, the makespan, the total weighted completion time, the total weighted discounted completion time, and the sum of the quadratic job completion times. We present heuristic algorithms by using the optimal permutations for the special cases of the corresponding single machine scheduling problems. We also analyze the worst-case bound of the proposed heuristic algorithms.



## 1. Introduction

In many branches of industry engineering and logistics management, there arise problems of ordering jobs on machines (Wang et al., 2010b; Li et al., 2011; Qi, 2011; Sun et al., 2012; Zhang and Tang, 2012; Cheng et al., 2012b; Janiak and Krysiak, 2012). In traditional scheduling problems, most research assumes that the production time of a given product is independent of its position in the production sequence. However, in many realistic settings, because firms and employees perform a task over and over, they learn how to perform more efficiently (i.e., performing setups, operating hardware and software, and handling raw materials and components). The production facility (a machine, a worker) improves continuously over time. As a result, the production time of a given product is shorter if it is scheduled later, rather than earlier in the sequence. This phenomenon is known as a "learning effect" in the literature (Badiru, 1992; Jaber and Bonney, 1999).

Biskup (1999) and Cheng and Wang (2000) were among the pioneers that brought the concept of learning into the field of scheduling, although it has been widely employed in management science since its discovery by Wright (1936). Biskup (1999) used the log-linear learning curve popularized in industrial engineering to describe the learning effect. He proved that single-machine scheduling with a learning effect remains polynomially solvable if the objective is to minimize the deviation from a common due date or to minimize the sum of flow time. Cheng and Wang (2000) considered a single machine scheduling problem in which the job processing times decrease as a result of learning. A volume-dependent piecewise linear processing time function was used to model the learning effects. The objective is to minimize the maximum lateness. They showed that the problem is NP-hard in the strong sense and then identified two special cases that are polynomially solvable. They also proposed two heuristics and analyzed their worst-case performance. Extensive surveys of different scheduling models and problems involving jobs with learning effects can be found in Biskup (2008) and in Janiak and Rudek (2009). More recent papers which have considered scheduling jobs with learning effects include Cheng et al. (2012a, 2008a, 2008b),Wang et al. (2008), Xu et al. (2008), Wu and Lee (2008), Janiak and Rudek (2008, 2009), Mosheiov (2008), Mosheiov and Sarig (2008), Janiak et al. (2009), Yin et al. (2009), Eren (2009), Cheng et al. (2009), Lee and Wu (2009), Wu and Liu (2010), Wang et al. (2010a), Wu et al. (2011), Wang and Wang (2011, 2012), Rudek (2011, in press), and Lee and Chung (2012). Cheng et al. (2008a) considered scheduling with learning effects in which the actual processing time of a job is a function of the total normal processing times of the jobs already processed and of the job's scheduled position. They proved that some single-machine and some flow shop scheduling are polynomially solvable. Cheng et al. (2008b) considered several scheduling problems with

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deteriorating jobs and learning effects. They showed that the single-machine problems are polynomially solvable if the performance criterion is makespan, total completion time, total weighted completion time, or maximum lateness. They also showed that the flow shop permutation problems are polynomially solvable under a certain condition. Xu et al. (2008) considered flow shop scheduling problems with a learning effect in which the actual processing times of jobs are defined as functions of their positions in a permutation. They presented algorithms for the following three regular performance criteria: the total weighted completion time, the discounted total weighted completion time, and the sum of the quadratic job completion times. Wu and Lee (2008) proposed a learning effect model where the actual job processing time not only depends on its scheduled position but also depends on the sum of the processing times of the jobs already processed. They showed that the single machine makespan and the total completion time problems are polynomially solvable under the proposed model and that the total weighted completion time has a polynomial optimal solution under certain agreeable conditions. Janiak and Rudek (2008, 2009) proposed a learning effect model where the actual job processing time not only depends on its scheduled position but also depends on the sum of the weighted processing times of the jobs already processed. Yin et al. (2009) considered some scheduling problems with general position-dependent and time-dependent learning effects. Eren (2009) considered a bi-criteria parallel machine scheduling problem with a learning effect of setup times and removal times. The objective function of the problem is minimization of the weighted sum of total completion time and total tardiness. He developed a mathematical programming model for the problem that belongs to the NP-hard class. Cheng et al. (2009) proposed a learning model where the actual job processing time is a function of the sum of the logarithm of the processing times of the jobs already processed. They showed that the problems to minimize the makespan and total completion time on a single machine are polynomially solvable under the proposed learning model. They also showed that the single machine scheduling problems to minimize the total weighted completion time, total tardiness, maximum lateness and maximum tardiness are polynomially solvable under some agreeable conditions on the problem parameters. Wang et al. (2010a, 2010b) considered single machine scheduling problems with exponential sum-of-logarithm-processing-times based learning effect. They proved that the makespan minimization problem, the total completion time minimization problem and the sum of the quadratic job completion times minimization problem can be solved by the SPT rule, respectively. Rudek (2011) considered flow shop scheduling with learning effect. He proved that the makespan minimization problem in the two-machine becomes at least NP-hard if job processing times are described by step functions dependent on a jobs position in a sequence. He also proved that the permutation version of the considered problem is strongly NP-hard if job processing times are described by stepwise learning models. He also constructed and proved optimality of the polynomial time algorithm that solves the weighted linear combination of sum and bottleneck linear assignment problems. Wang and Wang (2012) considered flow shop scheduling problems with an exponential learning effect. They presented heuristic algorithms for some regular objective functions. Bachman and Janiak (2004) proved that the single machine makespan minimization problem with ready times is strongly NP-hard with some position dependent linear learning models. They also claimed that this problem is still strongly NP-hard if job processing times are described by a non-increasing position dependent power function. However, Rudek (in press) proved that Bachman and Janiak (2004) showed NP-hardness only. Based on results provided by Rudek (in press), Janiak and Kovalyov (2012) proved that the problem analyzed by Bachman and Janiak (2004) is strongly NP-hard. Other types of jobs with learning effects have also been discussed; the reader is referred to papers by Mosheiov and Sarig (2008), Janiak et al. (2009), Lee and Wu (2009), Wu and Liu (2010), Wu et al. (2011), and Janiak and Kovalyov (2012).

In this paper we consider the same model as that of Kuo and Yang (2006) and Wang et al. (2008), but with flow shop scheduling. The time-dependent learning effect can be found in many real-life scheduling situations. The example is that firms and employees will learn more from performing jobs with longer processing times, i.e., the more time one has devoted to practising a skill, the better performance one will produce as a result of learning (Kuo and Yang, 2006). In this paper we will consider flow shop scheduling problems with a time-dependent learning effect. The objective is to minimize one of the five regular performance criteria namely, the total completion time, the makespan, the total weighted completion time, the total weighted discounted completion time, and the sum of the quadratic job completion times. We present a heuristic algorithm with worst-case bound for the special cases of the corresponding single machine problems.

The remaining part of this paper is organized as follows. In Section 2 we give some general notations and assumptions. In Section 3, we propose a heuristic algorithm with a worst-case bound for the total completion time minimization problem, makespan minimization problem, total weighted completion time minimization problem, total weighted discounted completion time minimization problem, and the sum of the quadratic job completion times minimization problem, respectively. In Section 4, we present the computational experiments. The last section contains some concluding remarks.

## 2. Notations and assumptions

The flow shop scheduling problem consists of scheduling $n$ jobs $J_1 J_2, \ldots J_n$ on $m$ machines $M_1, M_2, \ldots, M_m$. Each job $J_j$ consists of operations $(O_{1j}, O_{2j}, \ldots, O_{mj})$. Operation $O_{ij}$ has to be processed on machine $M_i, i=1,2,\ldots,m$. Processing of operation $O_{i+1,j}$ may start only after $O_{ij}$ has been completed and all machines process the jobs in the same order, i.e., a permutation schedule. The (normal) processing time of operation $O_{ij}$ is denoted by $p_{ij}$. The actual processing time $p_{ijr}$ of job $J_j$ on machine $M_i$ is a function dependent on its position $r$ in a schedule. As in Kuo and Yang (2006), in this paper, we consider a time-dependent learning effect model namely

$$p_{ijr} = p_{ij}\left(1+\sum_{l=1}^{r-1} p_{i[l]}\right)^{a}, \quad i=1,2,\ldots,m; \;\; r,j=1,2,\ldots,n, \tag{1}$$

where $a \le 0$ is the learning index.

For a given permutation $\pi$, $C_{ij} = C_{ij}(\pi)$ represents the completion time of operation $O_{ij}$, $C_j = C_{mj}$ represents the completion time of job $J_j$, $\pi = ([1],[2],\ldots,[n])$ represents a permutation of $(1,2,\ldots,n)$, where $[j]$ denotes the job that occupies the $j$th position in $\pi$, and $f(C) = f(C_1, C_2, \ldots, C_n)$ represents a regular measure of performance. We consider five regular objective functions namely, $\sum_{j=1}^n C_j$ (the total completion time), $C_{\max} = \max\{C_j | j=1,2,\ldots,n\}$ (the makespan), $\sum_{j=1}^n w_j C_j$ (the total weighted completion time, where $w_j > 0$ is a weight associated with job $J_j$), $\sum_{j=1}^n w_j(1-e^{-\gamma C_j})$ (the total weighted discounted completion time, where $\gamma \in (0,1)$ is the discount factor, see Pinedo, 2002; Section 3.1), and $\sum_{j=1}^n C_j^2$ (the sum of the quadratic job completion times, Townsend, 1978). In the remaining part of the paper, all the problems considered will be denoted using the three-field notation introduced by Graham et al. (1979).