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Order acceptance and scheduling in a two-machine flowshop

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ABSTRACT

We study the order acceptance and scheduling problem in a two-machine flowshop. The firm receives a pool of orders before a planning period, each of which is characterized by revenue, processing times on machines 1 and 2, a due date, and a tardiness penalty. The firm seeks to decide on the orders to accept and schedule the accepted orders so as to maximize the total net revenue. We formulate the problem as mixed-integer linear programming models, and develop a heuristic and a branch-and-bound (B&B) algorithm based on some derived dominance rules and relaxation techniques. We assess the performance of the B&B algorithm and the heuristic via computational experiments. The computational results show that the B&B algorithm can solve problem instances with up to 20 jobs within a reasonable time while the heuristic is efficient in approximately solving large instances of the problem.

1. Introduction

In today's competitive manufacturing environment, many firms increasingly adopt make-to-order production in order to provide customized services to satisfy the distinct requirements of customers. Customers typically require that the firm fulfills the due date promises or deadlines for their orders. Acceptance and processing of all the potential orders may not be a wise decision for the firm due to its limited production capacity, which may result in reduced revenue and even loss of customers. Selecting the proper orders to accept depends on the strategic direction of the firm and many other considerations. From a problem-oriented perspective, order acceptance should go along with analyzing capacity utilization. However, it is common in many industrial practices that the order acceptance and capacity planning decisions are made separately by the sales department and the production department, respectively. Naturally the sales department tends to accept as many orders as possible in order to maximize revenue while the production department primarily focuses on maximizing productivity. Such inter-departmental conflict of interest will result in considerable delay in order delivery or incurring extra resources. In order to maximize revenue, it is essential that firms consider order acceptance and production planning simultaneously.

The integrated order acceptance and scheduling problem has received increasing attention in recent years (see, e.g., Slotnick and Morton (2007), Rom and Slotnick (2009), Oğuz et al. (2010), Slotnick (2011), Talla Nobibon and Leus (2011), and Cesaret et al. (2012)). This stream of research has considered the problem involving different objective functions in various settings. These studies commonly assume that the orders are processed in the single-machine environment.

In this paper we consider the problem in the two-machine flowshop environment. Possessing distinct product characteristics, each order is described as a job with different processing times on machines 1 and 2. The two-machine flowshop model is motivated by many industries where the process to produce products typically comprises of two consecutive stages, e.g., a processing stage followed by a testing stage. An example is an equipment manufacturer who produces large special-purpose pressure-vessels. Each order includes only one type of pressure-vessels with distinct characteristics in terms of metal-material, size and shape, technological process standards, pressure performance index, and so on. It is common that processing a product is time consuming at one stage but not at the other, i.e., the bottleneck machine shifts with the processed orders. Under such circumstances, scheduling is an important issue. On the other hand, any delay in delivering an order beyond its due date may incur a penalty cost to the firm. Operating in such an environment, the firm faces the problem of order acceptance and scheduling in a two-machine flowshop to maximize the total net revenue.

The initial precursors to our study are Slotnick and Morton (1996) and Ghosh (1997). They consider the job acceptance and scheduling problem in the single-machine environment. They assume that the orders have static arrival times, deterministic processing times, weights that indicate customer priorities, and revenues, and they consider lateness penalties. The objective is to maximize the total net profit, which is the sum of the revenues of all the accepted orders minus any lateness penalties. Slotnick and Morton (1996) developed a branch-and-bound (B&B) algorithm and a couple of heuristics to solve the problem optimally and

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approximately. Ghosh (1997) focused on studying the theoretical aspects of the problem. Specifically, he established that the problem is NP-hard in the ordinary sense and provided a fully polynomial time approximation scheme (FPTAS). Lewis and Slotnick (2002) extended the problem to multiple periods for the case where rejecting a job of a customer will lead to the loss of all the future jobs from that customer. They developed a dynamic programming algorithm and a number of heuristics for this case. In fact, sequencing is not a computational burden for this case because there is a simple optimal sequence for the classical single-machine scheduling problem to minimize the total weighted lateness. Another variant of the problem is to find the optimal sequence of the accepted jobs to minimize the weighted tardiness. In recent years, Slotnick and Morton (2007) developed a B&B algorithm and heuristics to treat this variant of the problem. Rom and Slotnick (2009) developed genetic algorithms for the problem. Furthermore, Oğuz et al. (2010), Talla Nobibon and Leus (2011), and Cesaret et al. (2012) study other variants of the problem. Oğuz et al. (2010) and Cesaret et al. (2012) consider the case where the jobs have release times and sequence-dependent setup times. The former study gave a mixed-integer linear programming (MILP) model, which could be solved to optimality for instances with up to ten jobs within a one-hour time limit (also see Cesaret et al. (2012)), and developed three heuristics. The latter study mainly developed a tabu search algorithm. Talla Nobibon and Leus (2011) consider the case where there is an order pool that consists of two disjoint subsets of planning jobs and selectable jobs. They developed exact algorithms to solve this case. More details and extensive research results on this topic may be found in the recent reviews by Keskinocak and Tayur (2004), and Slotnick (2011).

This paper is organized as follows. In Section 2 we formally describe the problem under study. In Section 3 we construct two MILP models of the problem. In Section 4 we develop several heuristics and a B&B algorithm for the problem based on some dominance rules and relaxation techniques. In Section 5 we numerically evaluate the performance of the developed algorithms by running extensive computational experiments. In Section 6 we conclude the paper with a summary of the major research findings and suggest some future research directions.

2. Problem description

We formally describe the problem as follows: a set of jobs $N=\{1,2,\ldots,n\}$ is to be scheduled in a two-machine flowshop. All the jobs are available for processing at the beginning of the planning period. The processing times of each job i on machines 1 and 2 are p_{1i} and p_{2i}, respectively. Each machine can only process one job at a time and any job can begin processing on machine 2 only after completing its processing on machine 1. Associated with job i are its revenue u_i, due date d_i, and weight w_i that represents its unit time delay penalty beyond d_i in delivery to the customer. The decisions are to determine the jobs to accept for processing and to schedule the accepted jobs. The objective is to maximize the total net profit, which is the sum of the revenue of each accepted job minus its weighted tardiness.

3. Mixed-integer linear programming models

In this section we model the problem under consideration as two mixed-integer linear programming formulations, which can be solved by using the CPLEX software.

3.1. Formulation MILP1

The formulation is based on an optimal property of the problem, which is evident and stated without proof as follows:

Lemma 1. *For the accepted jobs, there is an optimal schedule in which each job is processed on both machines 1 and 2 in the same sequence.*

We define binary decision variables $y_i \in \{0,1\}$, $i=1,2,\ldots,n$, which take 1 if job i is accepted and 0 otherwise. From Lemma 1, we define binary variables $x_{ik}\in\{0,1\}$, $i,k=1,2,\ldots,n$, to identify the positions of the accepted jobs for processing on machines 1 and 2, which take 1 if job i is accepted and is the kth job processed on both machines 1 and 2, and 0 otherwise. We also introduce binary variables $z_{ij}\in\{0,1\}$, $i,j=1,2,\ldots,n$ and $i\neq j$, which are equal to 1 if and only if both jobs i and j are accepted and job j is processed on machines 1 and 2 in the position immediately after job i. In addition, we define real variables C_i and $T_i=\max\{0,C_i-d_i\}$, $i=1,2,\ldots,n$, as the completion time on the second machine and the tardiness of job i, respectively.

The above binary variables y_i and x_{ik} are adapted from Oğuz et al. (2010). In the two-machine flowshop, the completion time C_i of the current processed job i is determined not only by the completion time of the immediately preceding job i on machine 2, but also by the partial sequence of the other jobs preceding job i. Thus, it is necessary to define the additional binary variables z_{gh} to explicitly record the positions of the jobs preceding job i.

The MILP formulation is expressed as follows:

$$(\text{MILP1}):\quad \max \sum_{i=1}^{n}(y_i u_i - w_i T_i)$$

subject to

$$\sum_{k=1}^{n} x_{ik} = y_i \quad \forall i\in N, \tag{1}$$

$$\sum_{i=1}^{n} x_{ik} \le 1 \quad \forall k=1,2,\ldots,n, \tag{2}$$

$$z_{ij}\le y_i,\ z_{ij}\le y_j \quad \forall i,\ j\in N \text{ and } i\neq j, \tag{3}$$

$$x_{ik}+x_{j,k+1}\le z_{ij}+1 \qquad \forall i,\ j\in N,\ i\neq j \text{ and } k=1,2,\ldots,n-1, \tag{4}$$

$$C_i\ge 0 \quad \forall i\in N, \tag{5}$$

$$T_i\ge 0 \quad \forall i\in N, \tag{6}$$

$$C_i+y_j p_{2j}+(z_{ij}-1)M\le C_j \quad \forall i,\ j\in N \text{ and } i\neq j, \tag{7}$$

$$(p_{1j}+p_{2j})x_{j1}\le C_j \quad \forall j\in N, \tag{8}$$

$$\sum_{\ell=1}^{k}\sum_{i=1}^{n} p_{1i}x_{i\ell}+p_{1j}x_{j,k+1}+p_{2j}y_j+(y_j-1)M\le C_j+M\sum_{\ell=1}^{k}x_{j\ell} \quad \forall j\in N \text{ and } k=1,2,\ldots,n-1, \tag{9}$$

$$T_i\ge C_i-d_i \quad \forall i\in N, \tag{10}$$

$$\sum_{i=1}^{n} x_{ik}\ge \sum_{i=1}^{n} x_{i,k+1} \quad \forall k=1,2,\ldots,n-1. \tag{11}$$

In the above model, constraint (1) states that an accepted job is scheduled in exactly one position on both machines 1 and 2, while a rejected job is not put into any position. Constraint (2)