Int. J. Production Economics 141 (2013) 414–424

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Production lot sizing with a secondary outsourcing facility

Shine-Der Lee *, Shu-Chuan Lan

Department of Industrial and Information Management, National Cheng Kung University, 1 University Road, Tainan, Taiwan, ROC

ARTICLE INFO

Article history:
Received 14 October 2010
Accepted 28 August 2012
Available online 23 September 2012

Keywords:
Production lot sizing
Inventory model
Random demand
Outsourcing

ABSTRACT

An extended economic production quantity (EPQ) model under stochastic demand is investigated in this paper, where a fixed lot sizing policy is implemented to reduce the complexity of production planning and inventory control, and outsourcing with a secondary facility is used to supplement the lot sizing policy and to cope with the random demand. The considered cost includes: setup cost for the batch production, inventory carrying cost, backorder cost when the demand cannot be met immediately during the production period, and outsourcing cost when the total demand is greater than the lot size in one replenishment cycle. Under some mild conditions, the expected cost per unit time can be shown to be convex. Extensive computational tests have illustrated that the average cost reduction of the proposed model is significant when compared with that of the classical lot sizing policy. Significant cost savings can be achieved by deploying the production lot sizing policy with an outsourcing strategy when the mean demand rate is high.

1. Introduction

In the well-known Economic Production Quantity (EPQ) problem (Hadley and Whitin, 1963), one item is produced with constant production rate to meet the customer's demand, where the demand rate is assumed finite and constant. Each replenishment cycle in EPQ consists of two stages: the duration to produce a fixed lot size, and the duration to deplete remaining inventory before a new batch is launched. When the remaining inventory drops to zero, a new replenishment cycle will resume; that is, every replenishment cycle is identical. This fixed lot size policy is widely implemented for the production-manufacturing operations. More recently, Cárdenas-Barrón (2001) studied an EPQ model where the control policy is extended to determine both the maximum backorder level (amount of shortage or the lowest inventory level) prior to production and the maximum inventory level just after the production. Since demand is a deterministic constant, the amount of shortage is fixed at the maximum backorder level in every replenishment cycle, i.e., the system is active only when the inventory (negative level) drops to the maximum backorder level. Later, Ronald et al. (2004) developed an improved algebraic method to determine the two decision variables for this extended model. For a similar EPQ model, Sphicas (2006) found that if the backorder cost is relatively large, the classical EPQ is the optimal solution, i.e., the amount of backorder is small or negligible. When backorder cost is sufficiently small, it is more cost effective to consider the maximum backorder level. Pentico et al. (2009) further extended the above study to consider the case where a fixed percentage of shortage can be treated as lost sale.

Historically, the constant demand process of the production lot sizing policies has been extended to either a dynamic fashion or a stochastic fashion. For the first approach, termed dynamic lot sizing model, the entire planning horizon is consisted of n discrete periods ($t=1, 2, 3, \ldots, n$), where n is finite. Demand or order quantity from the customer is deterministic but can change over different periods. It seeks for an optimal production or procurement schedule, where the amount (lot size) of production or procurement at each period is to be determined. Since demand is not a constant in every period, the optimal solution of the lot size changes over the entire planning horizon. Wagner and Whitin (1958) first developed a polynomial time $O(n^2)$ algorithm that finds the optimal replenishment schedule, i.e., the lot size and the time for production. Later, Wagelmans et al. (1992) has improved the computational complexity to $O(n\log n)$ by using a geometrical interpretation and the time bounds. Kouvelis and Milner (2002) addressed a similar problem with non-stationary supply, and found that when the variance of demand increases, it is more cost effective to consider outsourcing strategy. Dynamic programming models were used in Chu et al. (2004) to determine the optimal lot sizing policy when shortages occur. Chu and Chu (2007) extended this study to consider storage space constraint when stock or inventory has been build up from the replenishment or the production.

For dynamic lot sizing models with time-varying demand, Donaldson (1977) established the optimal replenishment schedule in which the demand follows a linear trend with time. Later, Hariga (1993) developed an iterative heuristic to find the lot sizing policy with linear-trend demand. An iterative numerical

* Corresponding author. Tel.: +886 6 275 7575x53146; fax: +886 6 236 2162.
E-mail address: sdlee@mail.ncku.edu.tw (S.-D. Lee).

0925-5273/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ijpe.2012.09.008

procedure to compute the optimal replenishment schedule with linear or exponential time-varying demand is developed in Hariga and Benkherouf (1994). More recently, Goyal and Giri (2003) investigated a similar production-inventory problem in which both the production and the demand rates were assumed to vary with time, and a fixed fraction of shortage can be treated as lost sale. For dynamic lot sizing models with random demand, Axsäter (2011a) studied the choice of initial lot size policy when standard deviation of the demand per period is decreasing exponentially. Vargas and Metters (2011) adapted a dynamic lot sizing procedure to find the optimal replenishment schedule when demand is random and the planning horizon is finite.

Another stream of researches considers production lot sizing policies with random demand. Gavish and Graves (1980) formulated an up-to-level (s, S) inventory model, where demand is Poisson distributed. In this particular control policy, the production continues until the stock level reaches S; and a new lot size is resumed when the stock has depleted and fallen below s. That is, the lot size varies from one replenishment cycle (current batch) to the next one, depending on the random demand process and operational policy. Later, Graves and Keilson (1981) extended this (s, S) inventory model to the compound Poisson process. An up-to-level (S-1, S) production lot sizing policy is analyzed in Doshi et al. (1978) when demand follows a compound Poisson process. Partial backorder and a service measure are investigated in Rempala (2005), where the cost model in Gavish and Graves (1980) is extended. More recently, Axsäter (2011b) derived an optimal order up-to-level control policy where the demand is a compound Poisson process and no set-up or ordering cost is considered. Based on lot sizing policy, demand process, and shortage, these production lot sizing models are summarized in Table 1 below.

It is clear that randomness or fluctuation of demand has significant impacts on the lot sizing policy implemented in a production system. A varied or dynamic lot sizing policy is difficult to implement, due to the possible large fluctuation of workload on the production lines and high complexity of coordinating planning and control in a production system. A more cost effective production lot sizing policy is to maintain a fixed lot sizing policy in the primary production facility to reduce the operating cost, and utilize a secondary facility to handle the demand fluctuation to improve the service level. Greaver (1999), Kouvelis and Milner (2002), and Chu et al. (2004) suggested that outsourcing improves flexibility to meet changing business condition, demand for products, services and technologies. Harland et al. (2005) indicates that the circumstances in which these mixed policies might be appropriate have not been investigated. Many industries, such as notebook, furniture, and textile manufacturing, have outsourced parts of their production activities to secondary sectors, and shifted the focus from the product and process aspects to downstream supply chain oriented issues (Lutz and Ritter, 2009). These results indicated that a fixed production lot sizing policy, supplemented with subcontractors or outsourcing partners, provides flexibility and agility, and is cost effective for the operation management. This approach not only simplifies the production planning and inventory control, but also allows the factory to focus on their core competency.

This study is motivated by a case study in the manufacturing of notebook/netbook computer. Due to the rapidly changing market and short product life cycle, primary production facility is dedicated to assemble the products to meet the regular demand. Shortage caused by the fluctuation of market or demand is backordered and filled with a secondary production facility or outsourcing supplier. That is, a fixed lot sizing policy is deployed on the shop floor to simplify the production planning and inventory control, and to optimize the utilization of capital-intensive facility. A secondary facility, such as an outsourcing partner, is used to supplement the lot sizing policy to satisfy the orders, to cope with the random demand or changing market. The considered operating cost includes: setup cost to produce the lot size, inventory carrying cost, backorder cost when the demand cannot be satisfied immediately during the production period, and outsourcing cost if the total demand is greater than the lot size in one replenishment cycle. It is hence of great importance to study the production lot sizing policy with an outsourcing facility when faces random demand, to minimize the long run operating cost.

From Table 1, it is clear that the analysis in the current paper differs from the existing literature in at least three aspects. First, a fixed lot sizing policy is implemented in the proposed study, i.e., the lot size or the batch remains fixed over time; while the lot size

Table 1
A summary of the production lot sizing models in the literature.

Model type	Lot sizing policy	Demand process	Shortage	Related literature
Classical EPQ	Fixed lot size over time	Constant demand with infinite planning horizon	Not permitted	Hadley and Whitin (1963)
	Maximum backorder level and maximum inventory level		Backorder	Cárdenas-Barrón (2001), Ronald et al. (2004), Sphicas (2006)
			Partial backorder	Pentico et al. (2009)
Dynamic lot sizing	Varied lot size over time	Dynamic (Known demands or orders in n discrete periods with finite planning horizon)	Not permitted	Wagner and Whitin (1958), Wagelmans et al. (1992)
			Outsourcing	Kouvelis and Milner (2002)
			Backorder or outsourcing (independent)	Chu et al. (2004), Chu and Chu (2007)
		Time-varying demand with finite planning horizon	Not permitted	Donaldson (1977), Hariga (1993), Hariga and Benkherouf (1994)
			Backorder	Vargas and Metters (2011), Axsäter (2011a)
		Time-varying demand with infinite planning horizon	Partial backorder	Goyal and Giri (2003)
Stochastic lot sizing	Up-to-level (s, S) policy	Poisson process	Backorder	Gavish and Graves (1980)
		Poisson arrival with random demand quantity	Partial backorder	Doshi et al. (1978)
		Compound Poisson process	Backorder	Graves and Keilson (1981), Axsäter (2011b)
			Partial backorder	Rempala (2005)
	Fixed lot size over time	Poisson process	Mixture of backorder and outsourcing	The proposed study