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Use of Walsh transforms in estimation of depths of idealized sources from total-field magnetic anomalies ☆

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Abstract

We developed a scheme to compute the depths of four idealized magnetized sources, viz., a monopole, a line of monopoles, a dipole and a line of dipoles from the Walsh spectra of the total-field magnetic anomalies. The sequency numbers, l_{max} corresponding to the peaks of the differential energy density spectra over these sources are practically independent of the shape of the observed anomaly and are linearly dependent on the source depths. For each model, we derived a quantitative relation between the depth and the sequency number l_{max}. Analyses of simulated data over idealized isolated sources reveal that (i) a profile length of about 8 times the source depth provides accurate value in computed depth; (ii) data spacing of less than one-fourth the source depth has no significant error in depth computation and (iii) the technique is capable of tolerating random error to the tune of 10% of the peak amplitude of the simulated anomaly. We compared the results of depth estimations from Walsh and Fourier spectra. Analysis of total-field magnetic anomalies over a buried water supply pipe has demonstrated the applicability of the proposed method.

Keywords: Sequency number; Differential energy density; Shift-invariant

1. Introduction

Due to dipolar nature of magnetism, a magnetic anomaly map looks far more complex than a gravity map, posing considerable difficulties in quantitative interpretation. When the interpretation objective is either a regional study or large to moderate scale geological mapping, an interpreter remains satisfied with deriving qualitative information, e.g., delineating major trends, demarcation of different magneto-tectonic provinces, finding the presence of faults and shear zones, outlining intrusive bodies, etc. from the available magnetic maps. In exploration programs, on the other hand, the nature of magnetic anomaly interpretation varies from semiquantitative to quantitative. Whereas in hydrocarbon exploration, the main interest lies in mapping the basement, in mineral exploration, the problem consists of isolating individual anomalies and subsequent quantification of the sources in terms of their locations, depths, sizes and

☆Code available from server at http://www.iamg.org/CGEditor/index.htm.

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source geometries. An isolated magnetic anomaly is characterized by its amplitude, wavelength and shape or asymmetry. Whereas the wavelength of an anomaly is mainly dependent on the source geometry and depth, its shape or symmetry depends on the magnetization direction, orientation of the source, and direction of measurement besides the source geometry. The same source gives rise to different anomalies at different locations due to changes in the direction of the inducing magnetic field. This situation becomes further complicated in presence of remanent magnetization.

Several methods developed by previous workers attempt to reduce some of the above mentioned complexities. These methods include (i) reduction to pole (Baravov and Naudy, 1964); (ii) pseudo-gravity transformation (Baranov, 1957); (iii) separation of symmetrical and anti-symmetrical parts (Koulomzine et al., 1970); (iv) computation of analytic signal (Nabighian, 1972; Roest et al., 1992), etc. Of these methods, reduction to pole and pseudo-gravity transformation call for *a priori* information on the orientation of the source magnetization vector. A proper decomposition of an asymmetric anomaly into its even and odd components can be done *only if* the position of the source location is known. Although, for 2D sources, the magnitude of the analytic signal does not depend upon the orientation of the source magnetization vector and is therefore a potential tool for depth estimation, the same is not true for 3D bodies (Agarwal and Shaw, 1996; Li, 2006).

Walsh transforms have found several applications in exploration geophysics ranging from signal to noise ratio enhancement for archaeological site investigations using magnetic data (Gubbins et al., 1971), data compression for telemetry (Bois, 1972; Wood, 1974), processing of the marine seismic data (Chen, 1972; Chen and Boucher, 1973), identifying bed boundaries from well logs (Lanning and Johnson, 1983; Maiti and Tiwari, 2005), gravity anomaly interpretation (Shaw and Agarwal, 1990; Keating, 1992; Shaw et al., 1998), and resistivity mapping (Pal, 1991) and in global geophysics (Negi and Tiwari, 1990; Negi et al., 1993).

In the present work, we developed a simple scheme to estimate source depths from the Walsh spectra of the total-field magnetic anomalies over four idealized sources—a monopole, a line of monopoles, a dipole and a line of dipoles, etc. Fig. 1 shows geological settings that can be adequately described by these idealized sources.

We used the sequency number, $l_{\max}$ corresponding to the peak of the differential energy density (DED) spectrum computed from the modified Walsh spectra of the simulated anomalies to develop relations for depth estimation. We also investigated practical aspects of data analysis, viz., the effects of the profile length, sampling interval, padding of input data and random noise. Further, we compared the results with source depths estimated from conventional methods, e.g., using the slope of log-Fourier energy spectrum as a depth estimator. We evaluated the performance of the developed scheme through analysis of a real data over a buried water supply pipe to demonstrate the applicability of the proposed technique.

2. Walsh spectra

Walsh functions (Ahmed and Rao, 1975; Beauchamp, 1975) form a complete set of orthogonal functions having rectangular waveform. These functions assume values either +1 or −1. The concept of frequency does not apply to these functions because these are not truly periodic. To characterize such functions, Harmuth (1969) generalized the concept of frequency as half of the average number of zero crossings per unit interval and termed this as sequency. Ahmed and Rao (1975) have noted that Walsh transform gives rise to several spectral modes. They have also shown that few of these spectral modes are cyclic shift invariant. We have used one such cyclic shift invariant Walsh spectrum to develop the present technique. We describe below a brief account of this cyclic shift invariant Walsh spectrum.

Consider an N-periodic sequence $\{x(n)\}$ with $N = 2^v$, v being a positive integer. Let the corresponding Modified Walsh transform (MWT), be represented by Ahmed and Rao (1975, pp. 131–136)

$$\{x(n)\} \overset{\mathrm{MWT}}{\longleftrightarrow} \{X(k)\}. \tag{1}$$

The Walsh energy density spectrum then can be computed as

$$W_0(0) = X^2(0), \tag{2a}$$

$$W_m(0) = \sum_{k=2^{m-1}}^{2^m-1} X^2(k), \tag{2b}$$