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The effectiveness of a fuzzy mathematical programming approach for supply chain production planning with fuzzy demand

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ABSTRACT

The main focus of this work is to prove the effectiveness of a fuzzy mathematical programming approach to model a supply chain production planning problem with uncertainty in demand. A fuzzy optimization model that takes into account the lack of knowledge in market demand is developed. This work uses an approach of possibilistic programming. Such an approach makes it possible to model the epistemic uncertainty in demand that could be present in the supply chain production planning problems as triangular fuzzy numbers. The emphasis is on obtaining more knowledge about the impact of fuzzy programming on supply chain planning problems with uncertain demand.

1. Introduction

The concept of supply chain management (SCM), since their appearance in 1982 (see Oliver and Weber, 1982), is associated with a variety of meanings. In the eighties, SCM was originally used in the logistical literature to describe a new integrated approach of logistics management through different business functions (Houlihan, 1984). Then, this integrated approach was extended outside of the company limits to suppliers and customers (Christopher, 1992). In accordance with the Global Supply Chain Forum (Lambert and Cooper, 2000), the SCM is the integration of key business processes, from final users to original suppliers providing products, services and information which add value to clients, shareholders, etc. This paper is related to one of these key business processes: the supply chain production planning.

Supply chain production planning consists of the coordination and the integration of key business activities carried out from the procurement of raw materials to the distribution of finished products to the customer (Gupta and Maranas, 2003). Here, tactical models concerning mainly about inventory management and resource limitations are the focus of our work. In this context, with the objective of obtaining optimal solutions related to the minimization of costs, several authors have studied the modelling of supply chain planning processes through mathematical programming models (see, for instance, Alemany et al. (2009) and Mula et al. (2010)). However, the complex nature and dynamics of the relationships among the different actors of supply chains imply an important grade of uncertainty in the planning decisions (Bhatnagar and Sohal, 2005). Therefore, uncertainty is a main factor that can influence the effectiveness of the configuration and coordination of supply chains (Davis, 1993). One of the key sources of uncertainty in any production–distribution system is the product demand. Thus, demand uncertainty is propagated up and down along the supply chain affecting sensibly to its performance (Mula et al., 2005).

Along the years many researches and applications aimed to model the uncertainty in production planning problems (Mula et al., 2006a). Different stochastic modelling techniques have been successfully applied in supply chain production planning problems with randomness (Escudero, 1994; Gupta and Maranas, 2003; Sodhi and Tang, 2009). However, probability distributions derived from evidences recorded in the past are not always available or reliable. In these situations, the fuzzy set theory (Bellman and Zadeh, 1970) represents an attractive tool to support the production planning research when the dynamics of the manufacturing environment limit the specification of the model objectives, constraints and parameters. Uncertainty can be present as randomness, fuzziness and/or lack of knowledge or epistemic uncertainty (Dubois et al., 2002). Randomness comes from the random nature of events and deals with uncertainty regarding membership or non-membership of an element in a set. Fuzziness is related to flexible or fuzzy constraints modelled by fuzzy sets. Epistemic uncertainty is concerned with ill-known parameters modelled by fuzzy numbers in the setting of possibility theory (Dubois and Prade, 1988).

In this paper, for the purpose of demonstrating the usefulness and significance of the fuzzy mathematical programming for production planning, a fuzzy approach is applied to a supply chain production planning problem with lack of knowledge in demand

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data. The main contribution of this paper is an application of known possibilistic programming in a supply chain planning case study. Other applications of possibilistic programming in production planning problems can be found in Inuiguchi et al. (1994), Hsu and Wang (2001), Wang and Fang (2001), Lodwick and Bachman (2005), Wang and Liang (2005), Mula et al. (2008) and Vasant et al. (2008). However, previous researches mentioned above did not consider supply chain production planning problems.

This paper is organized as follows. Firstly, in Section 2, the supply chain production planning model, which has been the basis of this work, is described. In Section 3, a fuzzy model is developed to incorporate the demand uncertainty in the supply chain production planning model. Then, Section 4 uses a supply chain case study to illustrate the potential savings and other benefits that can be attained by using fuzzy models in a fuzzy environment. In Section 5, conclusions are given.

2. Description of the problem formulation

The mixed integer linear programming (MILP) model for supply chain production planning originally proposed by McDonald and Karimi (1997) is adopted as the basis for this work. The aim of this tactical model is to determine the sources of the limited resources of a company and the optimal assignment to its production resources to satisfy market demands at a minimum cost. The considered supply chain consists of multiple production facilities, globally located and producing multiple products. The demand of those products exists for a set of customers. The midterm planning horizon embraces from 1 to 2 years. Each production facility is characterized by one or more resources of semi-continuous production with limited capacity. The diverse products that are grouped in product families, in order to reduce transition times and costs between products of a family, compete for the limited capacity of those resources. This decision making process can be divided into two different phases: the production phase and the distribution phase or logistics. The production phase is focused on the efficient allocation of the production capacity in each one of the production plants with the objective of determining the optimal operative politics. In the distribution phase, they have considered the post-production activities like the demand fulfilment and the inventory management to satisfy the demand. Safety stock is kept to provide a buffer against uncertainty in demand. Finally, the structure of the supply chain can be classified as a network (Huang et al., 2003). Two layers of the supply chain network are considered: (1) manufacturing facilities and (2) customers. The production facilities can manufacture both finished products and intermediate from the raw materials. The intermediate products can be shipped to other production facilities where they are transformed into finished products which are subsequently shipped to customers.

Let us consider the following fuzzy formulation of the McDonald and Karimi's (1997) model. Decision variables and parameters for the mathematical programming model are defined in Table 1.

$$\text{Minimize } Z=\sum_{i,j,s,t} v_{ijs}P_{ijst}+\sum_{i,s,t} p_{is}C_{ist}+\sum_{i,s,t} h_{ist}I_{ist}+\sum_{i,s,c,t} t_{sc}S_{isct} + \sum_{i,s,s',t} t_{ss'}\sigma_{iss't}+\sum_{i,s,t}\zeta_{is}I^{\wedge}_{ist}+\sum_{i,c,t}\mu_{ic}I^{-}_{ict}+\sum_{f,j,s,t} f_{fjs}Y_{fjst} \tag{1}$$

Subject to

$$P_{ijst}=R_{ijst}RL_{ijst} \quad \forall i\in N^{RM}, \quad \forall j\in J, \quad \forall s\in S, \quad \forall t\in T \tag{2}$$

$$FRL_{fjst}\le H_{jst}Y_{fjst} \quad \forall f\in F, \quad \forall j\in J, \quad \forall s\in S, \quad \forall t\in T \tag{3}$$

Table 1
Decision variables and model parameters.

Sets of indices	
$T\equiv\{t\}$	Set of time periods
$I\equiv\{i\}$	Set of products. This set can be classified in raw materials, I^{RM}, intermediate products, I^{IP} and finished products, I^{FP}, so that $I=\{I^{RM}\cup I^{IP}\cup I^{FP}\}$. An intermediate product can also belong to the set of finished products
$F\equiv\{f\}$	Set of product families
$J\equiv\{j\}$	Set of resources
$S\equiv\{s\}$	Set of facilities
$C\equiv\{c\}$	Set of customers
Decision variables	
P_{ijst}	Quantity to produce product $i\in N^{RM}$ on resource j at site s in time period t
RL_{ijst}	Production time of product $i\in N^{RM}$ on resource j at site s in time period t
FRL_{fjst}	Production time of family f on resource j at site s in time period t
C_{ist}	Consumption of raw material or intermediate product $i\in N^{FP}$ at site s in time period t
I_{ist}	Inventory level of product $i\in N^{RM}$ at site s at the end of time period t
S_{isct}	Supply of finished product $i\in I^{FP}$ from site s to customer c in time period t
$\sigma_{iss't}$	Intermediate product flow $i\in I^{IP}$ from site s in time period t
I^{-}_{ict}	Shortage of finished product $i\in I^{FP}$ for customer c in time period t
$I^{\wedge}_{ist}$	Inventory deviation below safety stock target for product $i\in I$ at site s in time period t
Y_{ijst}	Binary variable which indicates if product I is produced on resource j at site s in time period t
Objective function cost coefficients	
μ_{ij}	Revenue per unit of product $i\in I^{FP}$ sold to customer c.
h_{ist}	Inventory cost of a unit of the product i at site s in time period t
p_{is}	Price of raw material $i\in I^{RM}$ at site s
ζ_{is}	Penalty for dipping below safety stock target of product i at site s
v_{ijs}	Variable cost of production of a unit of the product i on resource j at site s
$t_{ss'}\|t_{sc}$	Transportation cost to move a unit of product from site s to site s' or to customer c
f_{fjs}	Fixed cost of production for family f on resource j at site s
Technological coefficients	
R_{ijst}	Effective rate for product i using resource j at site s in time period t (it includes adjustment to the rate relating to efficiency, utility and/or yield)
$\beta_{i'is}$	Quantity of raw material or intermediate product $i\in N^{FP}$ that must be consumed to produce a unit of $i'\in N^{RM}$ at site s
General data	
k_{if}	0–1 parameter, which indicates if product i belong to family f
H_{jst}	Quantity of available time for production on resource j at site s in time period t
MRL_{fjs}	Minimum required time for family s on resource j at site s
$\tilde{d}_{ict}$	Fuzzy demand of finished product i for customer c in time period t
I^{L}_{ist}	Safety stock target for product i at site s in time period t
I_{is0}	Inventory of product i at site s at start of planning horizon

$$FRL_{fjst}\ge MRL_{fjs}Y_{fjst} \quad \forall f\in F, \quad \forall j\in J, \quad \forall s\in S, \quad \forall t\in T \tag{4}$$

$$FRL_{fjst}=\sum_{k_{if}=1} RL_{ijst}, \quad \forall f\in F, \quad \forall j\in J, \quad \forall s\in S, \quad \forall t\in T \tag{5}$$

$$\sum_{f} FRL_{fjst}\le H_{jst} \quad \forall j\in J, \quad \forall s\in S, \quad \forall t\in T \tag{6}$$

$$C_{ist}=\sum_{i'\ni\beta i'is\neq 0}\beta_{i'is}\sum_{j}P_{i'jst} \quad \forall i\in N^{FP}, \quad \forall j\in J, \quad \forall s\in S, \quad \forall t\in T \tag{7}$$

$$C_{ist}=\sum_{s'}\sigma_{is'st} \quad \forall i\in I^{IP}, \quad \forall s\in S, \quad \forall t\in T \tag{8}$$

$$I_{ist}=I_{is(t-1)}+\sum_{j}P_{ijst}-\sum_{s'}\sigma_{iss't}-\sum_{c}S_{isct} \quad \forall i\in N^{RM}, \quad \forall t\in T \tag{9}$$