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Pattern search algorithms for nonlinear inversion of high-frequency Rayleigh-wave dispersion curves

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Abstract

Inversion of Rayleigh wave dispersion curves is challenging for most local-search methods due to its high nonlinearity and to its multimodality. In this paper, we implemented and tested a Rayleigh wave dispersion curve inversion scheme based on GPS Positive Basis 2N, a commonly used pattern search algorithm. Incorporating complete poll and complete search strategies based on GPS Positive Basis 2N into the inverse procedure greatly enhances the performance of pattern search algorithms because the two steps can effectively locate the promising areas in the solution space containing the global minima and significantly reduce the computation cost, respectively.

The proposed inverse procedure was applied to nonlinear inversion of fundamental-mode Rayleigh wave dispersion curves for a near-surface shear (S)-wave velocity profile. The calculation efficiency and stability of the inversion scheme are tested on three synthetic models and a real example from a roadbed survey in Henan, China. Effects of the number of data points, the reduction of the frequency range of the considered dispersion curve, errors in P-wave velocities and density, the initial S-wave velocity profile as well as the number of layers and their thicknesses on inversion results are also investigated in the present study to further evaluate the performance of the proposed approach.

Results demonstrate that pattern search algorithms applied to nonlinear inversion of high-frequency surface wave data should be considered good not only in terms of accuracy but also in terms of the computation effort due to their global and deterministic search process.

Keywords: Pattern search algorithms; Generalized pattern search; Surface waves; Rayleigh waves; Dispersion curves

1. Introduction

In recent years Rayleigh waves have been used increasingly as a nondestructive tool to obtain near-surface S-wave velocity, one of the key parameters in environmental and engineering applications (Miller et al., 1999; Park et al., 1998, 1999; Xia et al., 1999, 2003; Foti et al., 2002; Beaty and Schmitt, 2003; Tian et al., 2003; Zhang and Chan, 2003; Lin and Chang, 2004; O'Neill et al., 2003; Ryden et al., 2004; Song et al., 2006). Utilization of surface wave dispersive properties may be roughly divided into three steps: field data acquisition, dispersion-curve picking, and inversion of phase velocities (Xia et al., 2004). Inversion of Rayleigh wave dispersion curves is one of the key steps in surface wave analysis to obtain a shallow subsurface S-wave velocity profile (Lai et al., 2002).

However, inversion of high-frequency Rayleigh wave dispersion curves, as with most other geophysical

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optimization problems, is typically a nonlinear, multiparameter, and multimodal inversion problem. Dal Moro et al. (2007) have demonstrated the high nonlinearity and multimodality by a synthetic model. Consequently, local optimization methods, e.g. matrix inversion, steepest descent, conjugate gradients, are prone to being trapped by local minima, and their success depends heavily on the choice of a good starting model (Boschetti et al., 1996) and the accuracy of the partial derivatives. Thus, global optimization methods that can overcome this limitation are particularly attractive for surface wave analysis (e.g., Meier and Rix, 1993; Yamanaka and Ishida, 1996; Beaty et al., 2002).

Pattern search algorithms are direct search methods well suitable for the global optimization of highly nonlinear, multiparameter, and multimodal objective functions (Lewis and Torczon, 1999). Pattern search algorithms have recently been used and tested to optimize complex mathematical problems characterized by the large numbers of local minima and/or maxima (Lewis and Torczon, 2000). However, few attempts have been made to address real-world geophysical problems, especially for the inversion of surface wave data. In this paper, the pattern search algorithm based on GPS Positive Basis 2N (Lewis and Torczon, 1999; Audet and Dennis, 2003) is implemented and tested on three synthetic models for a Rayleigh wave dispersion curve inversion and then effects of five different aspects involved in surface wave analysis on the performance of the algorithm are also investigated. Finally, we further verify the proposed inverse scheme using a real example from a roadbed site in Henan, China.

2. Fundamentals on pattern search algorithms

As described above, pattern search algorithms are direct search methods well capable of solving global optimization problems of irregular, multimodal objective functions, without the need of calculating any gradient or curvature information, especially for addressing problems for which the objective functions are not differentiable, stochastic, or even discontinuous (Torczon, 1997). As opposed to more traditional local optimization methods that use information about the gradient or partial derivatives to search for an optimal solution, pattern search algorithms compute a sequence of points that get closer and closer to the globally optimal point. At each iteration, the algorithms poll a set of points, called a *mesh*, around the current point—the point computed at the previous iteration of the algorithms, looking for a point whose objective function value is lower than the incumbent. If this occurs, the poll is called *successful* and the point they find becomes the current point at the next iteration. If the algorithms fail to find a point that improves the objective function, the poll is called *unsuccessful* and the current point stays the same at the next iteration. The mesh is formed by adding the current point to a scalar multiple (called *mesh size*) of a set of vectors (called a *pattern*). In addition to polling the mesh points, pattern search algorithms can perform an optional step at every iteration, called *search*. At each iteration, the search step applies another optimization method to the current point. If this search does not improve the current point, the poll step is performed (Lewis and Torczon, 2002).

Pattern search algorithms use the augmented Lagragian pattern search (ALPS) to solve nonlinearly constrained problems (Conn et al., 1991). The ALPS attempts to solve a nonlinear optimization problem with nonlinear constraints, linear constraints, and bounds using Lagrange multiplier estimates and penalty parameters (Lewis and Torczon, 2002). ALPS begins by using an initial penalty parameter and a starting point $\mathbf{X}_0$ for starting the optimization process. Pattern search algorithms minimize a sequence of subproblems, which are approximations of the original problem. When the subproblems are minimized to a required accuracy and satisfy feasibility conditions, the Lagrangian estimates are updated. Otherwise, the penalty parameter is increased by a penalty factor. This results in a new subproblem formulation and a minimization problem. These steps are repeated until the stopping criteria are met. For a more detailed description of the algorithms, the interested reader can refer to several excellent publications that extensively cover the subject (e.g., Conn et al., 1991; Torczon, 1997; Lewis and Torczon, 1999, 2000, 2002; Audet and Dennis, 2003; Kolda et al., 2003).

3. Pattern search algorithms for surface wave analysis

We implemented a series of MATLAB tools based on MATLAB 7.1 for surface wave analysis. We have worked on SWIPSA, a software package for surface wave inversion via pattern search