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Finite element modelling of the effective elastic properties of partially saturated rocks

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Abstract

Simulation of effective physical properties from microtomographic 3D images of porous structures allows one to relate properties of rocks directly to their microstructure. A static FEM code has been previously used to estimate effective elastic properties of fully saturated monomineralic (quartz) rock under wet and dry conditions. We use the code to calculate elastic properties under partially saturated conditions. The numerical predictions are compared to the Gassmann theory combined with Wood's formula (GW) for a mixture of pore fluids, which is exact for a monomineralic macroscopically homogeneous porous medium.

Results of the numerical simulations performed for two Boolean sphere pack distributions show significant deviation from the GW limit and depend on the spatial distribution of fluids. This is shown to be a numerical artefact caused by incomplete equilibration of fluid pressure, which is primarily due to insufficient spatial resolution.

To investigate the effect of pore-size and pore geometry, we perform FEM simulations for a model with regular pore geometry, where all pore channels have the same size and shape. Accuracy of these simulations increases with the total cross-section area of the channels and the size of individual channels. For the case where the total cross-section of the channels is large enough (on the same order as total porosity), there is a minimum of 4 voxels per channel diameter required for adequate fluid pressure equilibration throughout the pore space. Increasing the spatial resolution of the digital models reduces the discrepancy between the simulations and theory, but unfortunately increases the memory and CPU requirements of the simulations.

Keywords: Poroelasticity; Partial saturation; Effective elastic properties of rocks; Finite element method; Gassmann theory

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1. Introduction

Quantitative interpretation of geophysical data requires the knowledge of relationships between physical properties of rocks and their microstructure.

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With the advances in computer technology, it has become feasible to compute macroscopic physical properties of porous rocks from its microstructural information using rigorous numerical simulations of physical experiments in realistic pore space geometries. This approach, which has become known as digital (or computational) rock physics, has been used to model the effect of pores, fractures and fluids on the effective acoustic properties (Arns et al., 2002; Grechka, 2003; Gurevich et al., 2005; Roberts and Garboczi, 2000; Saenger et al., 2004) as well as geometrical, hydraulic and electric properties of rocks (Arns et al., 2001, 2004; Auzerais et al., 1996; Keehm et al., 2004; Knackstedt et al., 2005; Schwartz et al., 1994; Spanne et al., 1994). The computational rock physics technology relies on digitalized microstructural information which can be obtained either through X-ray microtomography (Dunsmuir et al., 1991; Flannery et al., 1987; Spanne et al., 1994), laser confocal microscopy (Fredrich et al., 1995), or stochastic models (Adler et al., 1992; Arns, 2002; Roberts and Garboczi, 2002; Yeong and Torquato, 1998). The numerical approach allows one to study properties of complex multi-phase materials with physically realistic phase distributions. However, computer simulations are not exact and their accuracy is limited by the simulation algorithm and by computational parameters such as spatial discretization. Therefore, in order to optimize the computational parameters, it is important to test the results of such simulations against known rigorous solutions.

One algorithm that has proved effective in simulations of the linear elastic properties of rocks is a finite-element method or FEM (Arns et al., 2002; Garboczi, 1998). There is a difficulty in testing such algorithms as values of the elastic properties are not known exactly for a porous material of any particular 3-D pore geometry. However, one rigorous theory applicable to the elastic properties in such a case is the Gassmann theory (1951), which is exact for a monomineralic macroscopically homogeneous porous medium with fully connected statistically isotropic pore space and no chemical/physical interaction between rock and pore fluid. In this paper, we propose a test scenario for static numerical simulations for a medium saturated with a fluid mixture. This test scenario is based on application of the Gassmann theory. When the Gassmann theory is applied to a medium saturated with a fluid mixture, the effective bulk modulus of a mixture is computed with Wood's equation (1955), which is an exact isostress formula for a mixture of Newtonian fluids. Wood's equation implies that fluid pressure is spatially uniform throughout the pore space. Since all the assumptions of the Gassmann theory could be simulated by FEM, a comparison of the numerically predicted parameters with the theory provides a natural test of the accuracy of numerical results. Arns et al. (2002) have shown that FEM simulations on media saturated with one fluid agree with the Gassmann predictions for a variety of porous structures. Here we extend those simulations to porous rocks saturated by a mixture of two fluids and test the predictions against Gassmann solutions. The goal is to determine computational parameters that ensure accurate and robust simulations.

The paper is organized as follows. Section 2 gives a brief outline of FEM used to derive the linear elastic properties. Section 3 describes the test scenario of the accuracy of the numerical simulations. Section 4 deals with numerical predictions of the effective elastic moduli. First, we perform the numerical experiments for Boolean random models of quartz saturated with gas and water. Then Wood's formula is verified for models of a fluid mixture (no solid phase). In the next set of numerical experiments, we focus on models of partially saturated idealized materials with regular geometries. We investigate the effect of pore channel size and the total area of water–gas interface on numerical predictions. Finally, we examine the effect of spatial resolution of the images on the FEM code performance. Section 5 summarizes our conclusions.

2. Numerical method

A method particularly suitable for the prediction of linear effective elastic properties of porous rocks from its microstructure is FEM by Garboczi (1998) and Arns et al. (2002). FEM is specially designed to handle arbitrary voxel-based models. Given bulk and shear moduli of material solid constituents and fluid phases, FEM can simulate the macroscopic physical properties, such as the effective elastic modulus, on 3D images of porous rocks at the pore scale. The method uses a variational formulation of the static linear elastic equations, and finds the solution by minimizing the elastic energy using a fast conjugate-gradient solver. A macroscopic strain is applied, with the resultant average stress or the average elastic energy giving the effective elastic