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A stochastic model to study the system capacity for supply chains in terms of minimal cuts

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ABSTRACT

For a single-commodity stochastic flow network, the system capacity is the maximum flow from the source to the sink. We construct a *p*-commodity stochastic flow network with unreliable nodes, in which branches and nodes all have several possible capacities and may fail, to model a supply chain. Different types of commodities, transmitted through the same network simultaneously, consume the capacities of branches and nodes differently. That is, the capacity weight depends on branches, nodes and types of commodity. We first define the system capacity as a vector and propose a performance index, the probability that the upper bound of the system capacity is a given pattern. Such a performance index can be easily computed in terms of upper boundary states meeting the demand exactly. An efficient algorithm based on minimal cuts is thus presented to generate all upper boundary states. The manager can apply this performance index to measure the transportation level of a supply chain.

1. Introduction

It is a crucial issue for a manager to derive the current system capacity (or residual system capacity) for a supply-demand system in order to respond quickly whether the customer's order can be accepted or not. In a binary-state network (without flow through it) with unreliable nodes, the branches and nodes are all in failure or operational state. Aggarwal et al. (1975) proposed a concept in which the failure of a node implies the failure of branches incident from it. Then the original network with unreliable nodes can be modified to a conventional network with perfect nodes. However, this concept is only adopted for a binary-state network. The system capacity has only values 0 and 1 to represent failure and operational state, respectively.

In a single-commodity binary-state flow network, each branch has a designated capacity which will have zero level only due to any failure. The system capacity is the maximum flow from the source to the sink. However, the branch should be stochastic due to that branch may be in failure, maintenance, partially reserved by other customers, etc. Then the network is thus called a single-commodity stochastic flow network. Since its system capacity is not a fixed number, several authors (Lin et al., 1995; Lin, 1998; Xue, 1985; Yeh, 1998) presented algorithms to evaluate the system reliability, the probability that the lower bound of the system capacity equals the demand d, for perfect nodes case. Jane et al. (1993), Lin (2001a, 2007b, c) and Yeh (2001) used minimal cuts (MCs) to generate upper boundary states for d in order to evaluate the system unreliability, the probability that the upper bound of the system capacity equals the demand d, for perfect nodes case. A MC is a set of branches whose proper subsets are no longer cuts, and an upper boundary state for d is a maximal system state meeting the demand d exactly. Lin (2002) extended the unreliability problem to the unreliable nodes case. In practice, the system reliability and unreliability are appropriate performance indices to measure the quality level of a single-commodity stochastic flow network.

Moreover, in a p-commodity stochastic flow network, multiple types of commodity are transmitting through the same network simultaneously where p is the number of types of commodity. In the past few decades, many researchers (Assad, 1978; Ford and Fulkerson, 1974; Held et al., 1974; Hu, 1963; Jarvis, 1969; Rothechild and Whinston, 1966) have solved the p-commodity maximum flow problem to find the maximal total flow under the assumption that each branch is deterministic. However, the maximal total flow is not a suitable performance index in case different type of commodity consumes the branch capacity differently. For instance, as the data shown in Table 1, the total flow in network 1 is larger than that in network 2. But this fact does not imply that network 1 provides a better transmission ability if commodity 2 consumes more capacity than commodity 1 does.

In this paper we concentrate on a p-commodity stochastic flow network with unreliable nodes (named a PMFUN throughout this paper) to study on the system capacity for a supply chain. Especially, the consumed capacity by different type of commodity varies with branches and nodes. In other words, the capacity weight depends on commodity type and network components

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Table 1
The total flow for two networks.

	Network 1	Network 2
Flow of commodity 1	5	2
Flow of commodity 2	4	6
Total flow	9	8

(branches or nodes). Many real-world systems such as logistics systems, computer systems, and telecommunication systems can be modeled as a PMFUN. Bailey and Francis (2008) presented case-study-based evidence to manage information flows for improved value chain performance. This paper is organized as follows. The system capacity will be discussed and defined in Section 3. Then a new performance index, the probability that the upper bound of the system capacity equals a given pattern, is proposed. Such an index can be easily derived in terms of upper boundary states for (d_1, d_2,…,d_p), where d_k the required quantity of commodity k at the sink t, k=1, 2,…,p. An algorithm based on MCs to generate all upper boundary states for (d_1, d_2,…, d_p) is presented in Section 4. Time complexity and storage complexity of the proposed algorithm are both analyzed. A container transportation example is shown in Section 5 to illustrate the proposed algorithm and how to compute the performance index.

2. Assumptions

Let $G\equiv(B, Q, M, W)$ denote a PMFUN where $B\equiv\{b_i|1\le i\le n\}$ the set of branches, $Q\equiv\{b_i|n+1\le i\le n+q\}$ the set of nodes except for the source s and the sink t, $M\equiv(M_1, M_2,\ldots, M_{n+q})$ with M_i the maximal capacity of b_i and $W\equiv\{w_i^k|i=1, 2,\ldots, n+q, k=1, 2,\ldots, p\}$ with w_i^k the capacity weight denoting the consumed capacity on b_i per commodity k. The current capacity of the component b_i is denoted by x_i and the vector $X\equiv(x_1, x_2,\ldots, x_{n+q})$ is called the system state. The PMFUN is required to further satisfy the following assumptions:

1. The branches and nodes all have several possible capacities and may fail.
2. All p types of commodity are transmitted from s to t.
3. The capacities of different components are statistically independent.
4. The current capacity x_i takes values from $\{0, 1, 2,\ldots, M_i\}$, i=1, 2, …, $n+q$.
5. The flows in G must satisfy the flow-conservation law (Ford and Fulkerson, 1962).

Vectors operations are done according to the following rules:

$X\le Y$ $(x_1, x_2,\ldots, x_{n+q})\le(y_1, y_2,\ldots, y_{n+q})$: $x_i\le y_i$ for $i=1, 2,\ldots, n+q$
$X<Y$ $(x_1, x_2,\ldots, x_{n+q})<(y_1, y_2,\ldots, y_{n+q})$: $X\le Y$ and $x_i<y_i$ for at least one i
$(d_1, d_2,\ldots, d_p)\le(d_1', d_2',\ldots, d_p')$: $d_k\le d_k'$ for k=1, 2,…, p
$(d_1, d_2,\ldots, d_p)<(d_1', d_2',\ldots, d_p')$: $(d_1, d_2,\ldots, d_p)\le(d_1', d_2',\ldots, d_p')$ & $d_k<d_k'$ for at least one k
$(d_1, d_2,\ldots, d_p)+(d_1', d_2',\ldots, d_p')$: $(d_1+d_1', d_2+d_2',\ldots, d_p+d_p')$

3. A PMFUN model

Since the nodes can fail, we redefine a cut as a set of branches and nodes the removal of which will disconnect s and t. Then a MC is also redefined as a set of branches and nodes whose proper subsets are no longer cuts. Suppose that $K_1, K_2,\ldots, K_m$ are m MCs. With respect to each MC $K_r=\{b_{r1},b_{r2},\ldots,b_{rn_r}\}$ where n_r is the number of components in K_r, the vector $F_r=(F_r^1,F_r^2,\ldots,F_r^p)$ is called a flow vector where $F_r^k=(f_{r1}^k,f_{r2}^k,\ldots,f_{rn_r}^k)$ with f_{rj}^k denoting the flow of commodity k through b_{rj}, j=1, 2,…, n_r, k=1, 2,…, p. The vector F_r is feasible under the system state X=(x_1, x_2,…, x_{n+q}) if

$$\sum_{k=1}^{p} w_{rj}^k f_{rj}^k\le x_{rj}\text{for } j=1,2,\ldots,n_r. \tag{1}$$

This inequality says that the total quantity $\sum_{k=1}^{p} w_{rj}^k f_{rj}^k$ of capacity on b_{rj} consumed by all types of commodities cannot exceed the current capacity x_{rj}.

Under the system state X, the MC K_r is said to support the demand (d_1, d_2,…, d_p) if there exists an F_r feasible under X such that

$$\sum_{j=1}^{n_r} f_{rj}^k=d_k\text{for } k=1,2,\ldots,p. \tag{2}$$

Under X, K_r is said to support at most (d_1, d_2,…,d_p) (i.e., K_r supports (d_1, d_2,…,d_p) but cannot support any (d_1', d_2',…,d_p') with (d_1', d_2',…,d_p')>(d_1, d_2,…,d_p)) if K_r supports (d_1, d_2,…,d_p) and there exists no F_r feasible under X such that

$$\begin{cases}\sum_{j=1}^{n_r} f_{rj}^1=d_1+1, & \text{for } k=1\\ \sum_{j=1}^{n_r} f_{rj}^k=d_k, & \text{for } k=2,3,\ldots,p.\end{cases} \tag{3}$$

Note that K_r supports (d_1+1, d_2,…,d_p) if F_r satisfies Eq. (3).

The system state X is said to support (d_1, d_2,…, d_p) if under X, all MCs support (d_1, d_2,…, d_p). Furthermore, X is said to support at most (d_1, d_2,…, d_p) if under X, all MCs support (d_1, d_2,…, d_p) and at least one MC supports at most (d_1, d_2,…, d_p). In a PMFUN, the system capacity $T(X)$ under X is defined to be (d_1, d_2,…, d_p) if X supports at most (d_1, d_2,…, d_p).

3.1. Upper boundary states for (d_1, d_2,…, d_p)

Similar to single-commodity case, the performance indices: $L_{d_1,d_2,\ldots,d_p}=Pr\{T(X)\ge(d_1,d_2,\ldots,d_p)\}$ and $U_{d_1,d_2,\ldots,d_p}=Pr\{T(X)\le(d_1,d_2,\ldots,d_p)\}$ can be adopted to evaluate the quality level of a PMFUN. The former is the probability that the lower bound of the system capacity equals (d_1, d_2,…, d_p) and the latter is the probability that the upper bound of the system capacity equals (d_1, d_2,…, d_p). Lin (2001b) had proposed an algorithm in terms of minimal paths to evaluate $L_{d_1,d_2,\ldots,d_p}$ for perfect nodes case. A minimal path is a path from the source to the sink without cycles. We focus on $U_{d_1,d_2,\ldots,d_p}$ for a PMFUN in terms of minimal cuts. However, it is not a wise way to enumerate all X such that $T(X)\le(d_1, d_2,\ldots, d_p)$ if the network is large. It will be more efficient to compute $U_{d_1,d_2,\ldots,d_p}$ if all maximal vectors in $\{X|T(X)\le(d_1, d_2,\ldots, d_p)\}$ can be found in advance.

We define the system state X as an upper boundary state for (d_1, d_2,…, d_p) if (i) $T(X)$=(d_1, d_2,…, d_p) and (ii) $T(Y)$>(d_1, d_2,…, d_p) for any system state Y with $Y>X$. Hence, the set of upper boundary states for (d_1, d_2,…, d_p) is the set of maximal ones in $\{X|T(X)=(d_1, d_2,\ldots, d_p)\}$ and also the set of maximal ones in $\{X|T(X)\le(d_1, d_2,\ldots, d_p)\}$. Thus, we have the following simple result:

Lemma 1. *Each system state X less than any upper boundary state for* (d_1, d_2,…, d_p) *supports at most* (d_1, d_2,…, d_p).

3.2. Theory

The remainder work for solving the system capacity problem is how to generate all upper boundary states for (d_1, d_2,…, d_p).