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Numerical modeling of ground-penetrating radar in 2-D using MATLAB ☆

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Abstract

We present MATLAB codes for finite-difference time-domain (FDTD) modeling of ground-penetrating radar (GPR) in two dimensions. Surface-based reflection GPR is modeled using a transverse magnetic (TM-) mode formulation. Crosshole and vertical radar profiling (VRP) geometries are modeled using a transverse electric (TE-) mode formulation. Matrix notation is used in the codes wherever possible to optimize them for speed in the MATLAB environment. To absorb waves at the edges of the modeling grid, we implement perfectly matched layer (PML) absorbing boundaries. Although our codes are two-dimensional and do not incorporate features such as dispersion in electrical properties, they capture many of the important elements of GPR surveying and run at a fraction of the computational cost of more elaborate algorithms. In addition, the codes are well commented, relatively easy to understand, and can be easily modified for the user's specific purpose.

Keywords: Ground-penetrating radar; GPR; Numerical modeling; Electromagnetics; Finite-difference time domain; FDTD

1. Introduction

Ground-penetrating radar (GPR) is a popular geophysical method for high-resolution imaging of the shallow subsurface. The GPR technique can be divided into two main modes of operation: (i) surface-based reflection surveying, where the transmitter and receiver antennas are located on the surface of the earth and the subsurface is imaged in terms of changes in its electrical properties, and (ii) borehole surveying, where one or both antennas are located in boreholes and subsurface properties are estimated tomographically. Of interest in our research is the application of both surface and borehole GPR to hydrogeological problems. Specifically, we are interested in using these techniques to assist in the development of hydrogeological models that predict groundwater flow and contaminant transport. A critical step in using GPR for this purpose is determining the link between the hydrogeological properties that govern these processes, and the information contained in a GPR data set.

Numerical GPR models provide one means of exploring the link between subsurface properties and GPR data. We can create a model of a subsurface region of interest, where we define the

☆ Code available from server at http://www.iamg.org/CGEditor/index.htm

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subsurface in terms of its lithological or hydrogeological properties. We can then transform this model into one that represents the subsurface in terms of its electrical properties. GPR modeling can then be used to simulate the acquisition of data in this subsurface region. The synthetic data that are obtained can be used to advance our understanding of the way in which information about the spatial variability of subsurface properties is captured by, and can be extracted from, GPR data.

A number of approaches have been presented for the numerical modeling of GPR data. These include ray-based methods (Goodman, 1994; Cai and McMechan, 1995), frequency-domain methods (Powers and Olhoeft, 1994; Zeng et al., 1995), integral methods (Ellefsen, 1999), and pseudospectral methods (Carcione, 1996; Casper and Kung, 1996; Lui and Fan, 1999). What has become by far the most common approach for GPR modeling over the past decade, however, is the finite-difference time-domain (FDTD) technique (e.g., Wang and Tripp, 1996; Bourgeois and Smith, 1996; Bergmann et al., 1996; Teixeira et al., 1998; Holliger and Bergman, 2002). Reasons for this include that the FDTD approach is relatively conceptually simple, accurate for arbitrarily complex models, and capable of accommodating realistic antenna designs and features such as dispersion in electrical properties (Taflove, 1995). What is lacking, however, are FDTD modeling codes for GPR, freely available for the public use, that are easy to understand and modify.

Here, we present FDTD codes, written in the MATLAB programming language, for basic surface reflection and borehole GPR modeling in two dimensions. Although 2-D modeling is limited in the sense that it cannot fully account for antenna behavior and out-of-plane variations in material properties, our codes capture many of the important features of GPR surveying and run at a fraction of the computational cost of fully 3-D algorithms. The codes feature perfectly matched layer (PML) absorbing boundaries to avoid reflections from the edges of the modeling grid. To optimize the programs for speed in MATLAB, matrix notation is used wherever possible. To begin, we discuss the theory behind our codes, including the governing analytical equations, their finite-difference approximations, numerical stability and dispersion criteria, and boundary conditions. Next, we briefly discuss how the FDTD equations are implemented in the MATLAB environment. Finally, we present two examples of the use of our codes, one showing modeling of a reflection GPR survey and the other modeling of a crosshole GPR survey.

2. Theory

2.1. Governing equations

We begin the theory behind our GPR modeling codes with Maxwell's curl equations in the frequency domain, which are

$$\nabla \times \mathbf{E} = -\mathrm{i}\omega\mu\mathbf{H}, \tag{1}$$

$$\nabla \times \mathbf{H} = \sigma\mathbf{E} + \mathrm{i}\omega\varepsilon\mathbf{E}, \tag{2}$$

where $\mathrm{i} = \sqrt{-1}$, ω is angular frequency, ε, μ, and σ are the dielectric permittivity, magnetic permeability, and electrical conductivity parameters, respectively, and $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field vectors. To implement PML absorbing boundaries in our codes, we consider the general case of a complex stretched coordinate space (e.g., Chew and Weedon, 1994; Gedney, 1998), where the ∇ operator takes the following form:

$$\nabla = \hat{x}\frac{1}{s_x}\frac{\partial}{\partial x} + \hat{y}\frac{1}{s_y}\frac{\partial}{\partial y} + \hat{z}\frac{1}{s_z}\frac{\partial}{\partial z}, \tag{3}$$

where

$$s_k = \kappa_k + \frac{\sigma_k}{\alpha_k + \mathrm{i}\omega\varepsilon_0}, \quad k = x, y, z \tag{4}$$

are complex coordinate stretching variables that vary only in the k direction (Kuzuoglu and Mittra, 1996). Here, ε_0 is the dielectric permittivity of free space, and σ_k, κ_k, and α_k are parameters that can be specified to allow for wave propagation in the interior of the modeling grid and wave absorption in the PML boundary regions. It should be stressed that σ_k, κ_k, and α_k are not true electrical properties. Rather they are parameters that, through complex coordinate stretching, add additional degrees of freedom to Maxwell's equations to allow for PML boundary implementation.

Taking the components of Eqs. (1) and (2) using the identity in Eq. (3), and assuming that there is no variation in the y direction for 2-D modeling, we arrive at the following two decoupled sets of partial-differential equations involving the $\{H_x, H_z, E_y\}$ and $\{E_x, E_z, H_y\}$ field components:

$$\mathrm{i}\omega\mu H_x = -\frac{1}{s_z}\frac{\partial E_y}{\partial z}, \tag{5a}$$