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# Pricing index-based catastrophe bonds: Part 1 Formulation and discretization issues using a numerical PDE approach

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ABSTRACT

This work is the first installment in a two-part series, and focuses on the development of a numerical PDE approach to price components of a Bermudan-style callable catastrophe (CAT) bond. The bond is based on two underlying stochastic variables; the PCS index which posts quarterly estimates of industry-wide hurricane losses as well as a single-factor CIR interest rate model for the three-month LIBOR. The aggregate PCS index is analogous to losses claimed under traditional reinsurance in that it is used to specify a reinsurance layer. The proposed CAT bond model contains a Bermudan-style call feature designed to allow the reinsurer to minimize their interest rate risk exposure on making substantial fixed coupon payments using capital from the reinsurance premium. Numerical PDE methods are the fundamental strategy for pricing early-exercise constraints, such as the Bermudan-style call feature, into contingent claim models. Therefore, the objective and unique contribution of this first installment in the two-part series is to develop a formulation and discretization strategy for the proposed CAT bond model utilizing a numerical PDE approach. Object-oriented code design is fundamental to the numerical methods used to aggregate the PCS index, and implement the call feature. Therefore, object-oriented design issues that relate specifically to the development of a numerical PDE approach for the component of the proposed CAT bond model that depends on the PCS index and LIBOR are described here. Formulation, numerical methods and code design issues that relate to aggregating the PCS index and introducing the call option are the subject of the companion paper.



## 1. Introduction

Earthquakes, hurricanes, tornadoes and hailstorms consist of the four most costly types of insured catastrophic perils in the United States. Of these, earthquakes and hurricanes pose the greatest catastrophic risk generating on average $9.7 billion in claims annually from 1989 through 2001 (GAO, 2002). Focusing specifically on hurricanes; Hurricanes Katrina, Wilma, Rita, Ophelia and Dennis caused $52.7 billion in insured losses in 2005 amounting to nearly 93% of all losses from catastrophic perils that year. In addition, Hurricanes Charley, Ivan, Frances and Jeanne produced $23 billion in insured losses in 2004. In contrast, Hurricane Andrew alone caused $15.5 billion in insured damages in 1992. Insurance companies often require reinsurance to limit their liabilities given the large capital requirements needed to cover these damages. Unfortunately, capital in the reinsurance industry is also limited relative to the magnitude of these damages, creating large fluctuations in the price and availability of reinsurance during years (and in those following) when catastrophic losses are excessive. In response, reinsurance companies have recently turned to the capital markets by issuing risk-linked securities in the form of catastrophe bonds (CAT bonds) to provide the collateral necessary for reinsurance. If a specified catastrophe does not occur (or if aggregate damages are less than a trigger level) before the maturity date of the bond, the investors get the full face value of the bond plus very generous coupon payments. If the specified catastrophe does occur (or if aggregate damages exceed the trigger level) before the maturity date, the bond defaults resulting in either a partial or no payment to investors. Fortunately, the capital markets are extremely large (approximately $31 trillion) relative to the scale of the property damage and can readily absorb this risk. Guy Carpenter & Company Inc. (2008) provides a 'year-end 2007 update' of all publicly disclosed transactions to date. They state that the CAT bond market has grown steadily when measured by the total outstanding risk capital of $13.8 billion at year-end 2007, compared to $8.48 billion at year-end 2006, $4.90 billion at year-end 2005, and $4.04 billion at year-end 2004. This total does not include private placements which are becoming more common. Cat bond risk principal now comprises 8% of the $169 billion property limits market globally, and 12% of the $81 billion United States market.

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Securitization of reinsurance into CAT bonds that ultimately get purchased by financial institutions (i.e. mutual, pension and hedge funds, etc.) implies that the pubic is directly exposed to financial losses due to insured catastrophic perils worldwide, beyond simply holding stock in a reinsurance or insurance company. Consequently, institutional investors need to combine knowledge from the disparate fields of geological and financial engineering in order to critically evaluate the risks disclosed in each CAT bond offering circular (Elsner et al., 2009). This paper is focused towards the professional geoscientist or engineer employed by an institutional investor to provide advice on how future expected industry-wide hurricane losses along the Gulf and Atlantic coasts of the United States are estimated and priced into CAT bonds based on observed meteorological conditions, historical property claims, and market conditions.

Pricing models for catastrophic risk-linked securities have followed two main financial methodologies: the theory of equilibrium pricing and the no-arbitrage valuation framework. Aase (1999), Cox and Pedersen (2000) and Cox et al. (2000) use the theory of equilibrium pricing while Sonderman (1991), Cox and Schwebach (1992), Cummins and Geman (1995), Geman and Yor (1997), Loubergé et al. (1999), Lee and Yu (2002), Dassios and Jang (2003), Romaniuk (2003), Burnecki and Kukla (2003), Cox et al. (2004), and Hardle and López Cabrera (2007) use the no-arbitrage framework. The primary inspiration for this research follows from Cummins and Geman (1995) who focused on pricing catastrophe insurance futures and call spreads which were traded on the Chicago Board of Trade between 1992 and 1999. The payout of these contracts was based on loss estimates for the insurance industry as a whole from a single, or multiple, catastrophe. The loss estimates were reported quarterly by Property Claims Services (PCS) as an index. The index is a function of losses claimed by a survey of insurers providing coverage within specific geographic regions of the United States, which in the case of hurricane damages, involves the Gulf and Atlantic coasts. Cummins and Geman (1995) claim the PCS index follows a stochastic process: namely, geometric Brownian motion with drift and jump diffusion. The two sources of randomness include volatility on the PCS index dues to ‘small’ catastrophes and delays in reporting claims from one quarter to the next, while jumps are due to ‘large’ catastrophes.

Although the CAT bond market has experienced significant innovation since its inception in the late nineties to today, in general the basic structure of all of the issued CAT bonds has followed a standard model. Kaplan and Lefebve (2003) provide a detailed description of this model which is summarized here. First, a reinsurer negotiates a reinsurance contract with a ceded insurer(s) and collects the appropriate premium. Next, the reinsurer establishes a captive offshore reinsurance special purpose vehicle (SPV) for legal, regulatory and tax reasons. The SPV is then used to issue the CAT bond to the capital markets and collect the proceeds from its sale. These proceeds are used as collateral to the full extent of the SPV's obligations via the reinsurer under the reinsurance contract, and are invested in high-grade debt securities. The SPV then enters a swap arrangement with an investment bank whereby interest from the high-grade debt securities is exchanged for LIBOR minus a slight swap spread to guarantee against default risk. Finally, the SPV then pays floating coupons in arrears to the bond holders quarterly to match the three-month LIBOR used in the swap. In addition, the swap spread plus additional fixed coupons are paid by the reinsurer, through the SPV, to the bond holders using capital from the reinsurance premium which the reinsurer retains. Payment of the floating coupons to the CAT bond holders is meant to eliminate interest rate risk on the CAT bond principal, while investment of the collateral in the high-grade debt securities plus the swap spread is meant to eliminate default risk. However, the reinsurer remains exposed to interest rate risk by paying the fixed coupons using capital from the reinsurance premium. To minimize this interest rate risk, the reinsurer retains the right to call the CAT bond for a specified call price. In response, prospective bond holds will devalue the purchase price of the CAT bond. Note that by exercising the call option, the reinsurer is making the choice to redeem the CAT bonds and use their own capital to cover the reinsurance layer.

Following the efforts of Cummins and Geman (1995) as well as Lee and Yu (2002), the objective of this work is to develop a no-arbitrage model for pricing CAT bonds which depend on industry-wide hurricane losses along the Gulf and Atlantic coasts of the United States as reported quarterly by the PCS index. The proposed model is abstracted from the general CAT bond structure described above in that the price is a function of the three-month LIBOR as well as the PCS index, and both floating and fixed coupons are paid quarterly. The objective and unique contribution of this work is to develop a control volume finite difference/element (i.e. numerical PDE) model for pricing a Bermudan-style (semi-American) call optionality into the pricing structure of the CAT bond. The proposed CAT bond model is then priced using literature-derived parameter values describing: the frequency and magnitude of industry-wide losses, the stochastic nature of the PCS index and LIBOR as well as parameters describing the contractual structure of the CAT bond. By pricing the CAT bond with and without the call feature, interest rate risk on the fixed coupon payments is valued. This risk is then compared to the sensitivity in the price of the CAT bond model to uncertainty in the frequency and magnitude of losses arising from hurricane events. In effect, the outcome is to determine the relative balance in financial risk exposure the issuer faces from market driven (LIBOR) and meteorological factors.

The objective of this research is presented over a sequence of two papers. The first paper focuses on issues pertaining to the formulation and discretization of a component of the proposed CAT bond model in a numerical PDE framework. In particular, the formulation of this component is based on two stochastic variables: quarterly losses as reported by the PCS index and the three-month LIBOR interest rate. The outcome of this first paper is to illustrate an object-oriented programming strategy to encapsulate all functions and variables pertaining to the numerical PDE solution of this component of the CAT bond model into a single object, called the CATbond $S$–$r$ object. The second paper then focuses on numerical and object-oriented programming issues needed to efficiently aggregate the PCS index each quarter and to introduce the call optionality by using multiple instances of the CATbond $S$–$r$ object. The payoff structure of the CAT bond is then specified on the aggregate PCS index to be reminiscent of a reinsurance layer. This completes the definition of the proposed CAT bond model. Finally, the second paper provides a sensitivity analysis examining the utility of the issuer being able to redeem the CAT bond and minimize their interest rate risk on the fixed coupon payments given uncertainty in the frequency and magnitude of losses arising from hurricane events.

## 2. Frequency, magnitude and reinsurance layers

The frequency and magnitude of catastrophic events are arguably the two most important parameters when engineering a CAT bond contract. There are two basic approaches to estimate these data. The first involves using numerical models to simulate the evolution and resulting damage from hurricanes, earthquakes, or other perils. These models are based on constitutive relationships