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Pricing index-based catastrophe bonds: Part 2
Object-oriented design issues and sensitivity analysis

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ABSTRACT

This work is the second installment in a two-part series, and focuses on object-oriented programming methods to implement an augmented-state variable approach to aggregate the PCS index and introduce the Bermudan-style call feature into the proposed CAT bond model. The PCS index is aggregated quarterly using a discrete Asian running-sum formulation. The resulting aggregate PCS index augmented-state variable is used to specify the payoff (principle) on the CAT bond based on reinsurance layers. The purpose of the Bermudan-style call option is to allow the reinsurer to minimize their interest rate risk exposure on making fixed coupon payments under prevailing interest rates. A sensitivity analysis is performed to determine the impact of uncertainty in the frequency and magnitude of hurricanes on the price of the CAT bond. Results indicate that while the CAT bond is highly sensitive to the natural variability in the frequency of landfalling hurricanes between El Ninõ and non-El Ninõ years, it remains relatively insensitive to uncertainty in the magnitude of damages. In addition, results indicate that the maximum price of the CAT bond is insensitive to whether it is engineered to cover low frequency high magnitude events in a 'high' reinsurance layer relative to high frequency low magnitude events in a 'low' reinsurance layer. Also, while it is possible for the reinsurer to minimize their interest rate risk exposure on the fixed coupon payments, the impact of this risk on the price of the CAT bond appears small relative to the natural variability in the CAT bond price, and consequently catastrophic risk, due to uncertainty in the frequency and magnitude of landfalling hurricanes.

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1. Introduction

An introductory review of the existing CAT bond pricing literature was provided in Unger (2009) as part of the first installment in this two-part series. This review focused on the growing use of catastrophe bonds as a supplement to standard reinsurance to cover insured losses arising from catastrophic perils. A CAT bond model was proposed as being dependant on loss estimates reported quarterly by Property Claims Services (PCS) as an index, where the loss estimates are due to hurricane damages along the Gulf and Atlantic coasts of the United States. The PCS index was described as following a stochastic process: namely, geometric Brownian motion with drift and jump diffusion. The two sources of randomness included volatility on the PCS index dues to 'small' catastrophes and delays in reporting claims from one quarter to the next, while jumps were due to 'large' catastrophes.

The component of the proposed CAT bond model that was introduced in the first installment (Unger, 2009) was based on elements of the standard CAT bond model issued by the reinsurance industry to date. In summary, the model pays both fixed and floating coupons to the prospective purchaser, where the value of the floating coupons is tied to the three-month LIBOR interest rate. Payment of the very generous fixed coupons is meant to entice prospective bond holders to purchase these risk-linked securities (with possible loss of principle), while the floating coupons are meant to eliminate interest rate risk on the CAT bond principle (to the issuer, and possibly the purchaser) due to stochastic fluctuations in the three-month LIBOR over the term of the bond. The issuer, however, remains exposed to interest rate risk from paying the fixed coupons to the prospective bond holders. To manage this risk, the issuer retains the right to call the CAT bond at prescribed notice dates for a specified call price. This call feature is characteristic of Bermudan-style callable bonds.

The objective of the first installment in this two-part series was to develop a numerical PDE (control-volume finite difference/element) approach to price the component of the proposed index-based CAT bond model that is based on the two stochastic variables: the PCS index and the three-month LIBOR. This resulted in the CATbond $S-r$ numerical PIDE component of the proposed CAT bond model. The outcome of this numerical PDE model was

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to provide a building-block on which to aggregate losses posted by the PCS index over the term of the CAT bond, and to introduce the early-exercise Bermudan-style call feature. Hence, the objective of this second installment is to focus on numerical and object-oriented programming issues needed to efficiently aggregate the PCS index and introduce the call feature using the CATbond $S-r$ numerical PDE object. Specifically, two additional space-like augmented-state variables are introduced in this work into the CAT bond model; with the first being used to aggregate the PCS index resulting in the CATbond $I-S-r$ model, and the second being used to introduce the call feature resulting in the CallableCATbond model. These augmented-state variables are not introduced into the proposed CAT bond model in a conventional numerical PDE manner; instead, they are implemented using object-oriented code design strategies centered around translating and modifying CAT bond price values obtained on multiple CATbond $S-r$ objects.

Following the complete development of the CAT bond model, the objective of this work is then to determine the sensitivity of the CAT bond price to uncertainty in parameters measuring the frequency and magnitude of insured losses due to hurricanes along the Gulf and Atlantic coasts of the United States as well as parameters involving the contractual structure of the CAT bond. These would include the term of the bond, the choice of reinsurance layers, and the rationale for invoking the call feature. Of specific interest is whether the impact of the Bermudan-style call feature on the price of the bond is significant in comparison to uncertainty in parameters measuring the frequency and magnitude of insured losses. In effect, the outcome is to determine the relative balance in financial risk exposure that the issuer faces from market-driven (LIBOR) and meteorological factors.

2. Augmented-state variables

The CATBond $S-r$ component of the proposed CAT bond model was introduced in Unger (2009) and represents the numerical PDE solution of $B(S,r,\tau)$ as a function of the two space-like variables S and r as well as the backwards time variable τ. The numerical PDE solution was formulated and discretized in a control-volume finite difference/element framework. To complete the development of the proposed CAT bond model, two additional space-like augmented-state variables need to be described. These augmented-state variables are not introduced into the CAT bond model in a conventional numerical PDE manner (i.e. see Zvan et al., 1999, as well as Vetzal and Forsyth, 1999); instead, they are implemented using an object-oriented code design where values of $B(S,r,\tau)$ on multiple numerical PDE objects (as discussed in Section 3.3 of Unger, 2009) are collectively modified at discrete times. The first augmented-state variable I is used to aggregate the PCS index variable S resulting in the CATbond $I-S-r$ model. The second augmented-state variable K is used to introduce the Bermudan-style call feature resulting in the CallableCATbond model. The formulation of the CATbond $I-S-r$ and CallableCATbond models completes the development of the proposed index-based CAT bond model, with implementation details being the subject of the remainder of this section.

Base-scenario CAT bond parameters are needed for the presentation of the CATbond $I-S-r$ and CallableCATbond models, and are provided in Table 1. The Bermudan-style call feature is represented by an additional series of constraints beyond the variables listed in Table 1. Hence, the base-scenario CAT bond does not explicitly contain the call feature. Note that these parameters are consistent with the discussion of frequency, magnitude and reinsurance layers provided in Section 2 of Unger (2009).

Table 1
Base-scenario CAT bond parameters.

Parameter	Value
General	
Maturity date T	1 year
Principal	\$1
Fixed coupon $\overline{C}$	2.0% per quarter
Lower reinsurance layer L_l	\$20 billion
Upper reinsurance layer L_u	\$30 billion
Catastrophic claim $C^{\mathbb{P}} = C^{\mathbb{Q}}$	\$40 billion
Catastrophic loss function	$g^{\mathbb{P}}(\eta C^{\mathbb{P}}) = g^{\mathbb{Q}}(\eta C^{\mathbb{Q}})$
PCS index S	
Damage appreciation rate α	0.1 per year
Market price of risk $\hat{q}_S$	-0.25 per year
Volatility σ_S	0.2
Event frequency $\lambda^{\mathbb{P}} = \lambda^{\mathbb{Q}}$	0.5 per year
CIR parameters for the three-month LIBOR r[a]	
Speed of adjustment κ	0.54958046
Market price of risk $\hat{q}_r$	-0.40663675
Reversion level θ_r	0.0348468515
Volatility σ_r	0.38757496

[a] Additional parameters can be found in Büttler (2002).

2.1. The CATbond $I-S-r$ model

The price of the CATbond $S-r$ component introduced in Unger (2009) is a function of the two-space like variables S and r. However, the proposed index-based CAT bond is a function of the augmented-state variable I which represents the aggregate PCS index. Introduction of this variable results in the CATbond $I-S-r$ model whose solution is denoted as $B(I,S,r,\tau)$. For hurricane damages along the Gulf and Atlantic coasts of the US, PCS posts industry-wide losses quarterly. For the CATbond $I-S-r$ model, these losses are aggregated at the end of each quarter. Within the context of financial engineering, I denotes the running sum of the stochastic variable S in an analogous manner to a discrete Asian option. The variable I is aggregated, and hence observed, at discrete (quarterly) times according to

$$I^n = \sum_{n=1}^{N} S^n \tag{1}$$

where I^n is the discrete running sum obtained by sampling S at n observation times $t_1, t_2, \ldots, t_N$ so that $S^n = S(t_n)$.

The payoff condition is specified along the I-axis given that this axis represents the aggregate PCS index and hence reinsurance layer. Following the earlier notation, the lower and upper reinsurance layers are given as L_l and L_u, respectively. Now, the payoff condition is

$$\beta = 1 \quad \text{for } I < L_l$$

$$\beta = \frac{L_u - I}{L_u - L_l} \quad \text{for } L_l < I < L_u$$

$$\beta = 0 \quad \text{for } I > L_u \tag{2}$$

where

$$B(I,S,r,\tau = 0) = \beta \times \$1 \tag{3}$$

This has the physical interpretation that for aggregate damages less than L_u, the CAT bond is not called upon to provide reinsurance coverage and hence does not default. For aggregate damages between L_l and L_u, a portion of the CAT bond is called upon to provide reinsurance coverage with all principal being exhausted at L_u. Beyond aggregate damages given by L_u, the CAT bond is worthless due to complete default. Parameters defining the payoff structure for the base-scenario contract are listed in Table 1 and are summarized here as: $L_l = \$20$ billion, $L_u = \$30$

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