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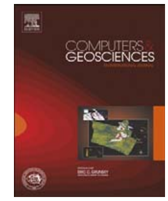

Analysis of kinematic correlations in faults and focal mechanisms with GIS and Fortran programs ☆

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ABSTRACT

The kinematic correlation between faults and focal mechanisms can be quantified using scalar (vector and tensor product) or geometric (rotation pole and angle) measures of the similarity of their orientations. The statistical properties of the correlation may help characterise the spatio-temporal properties of natural datasets and test their congruence with theoretical models. This paper describes GIS and Fortran 90/95 tools for analysing the kinematic correlation of faults, and for simulating fault movements in a homogeneous stress field. As an example, we analyse the Umbria-Marche 1997 seismic sequence with these tools; our results show a positive spatial correlation of seismic events that increases with time following the mainshocks.

1. Introduction

We can constrain the dynamic and mechanical factors that control the behaviour of earthquakes and faults by statistically characterising the spatial and temporal correlation between seismic events and faults in the area. Of the many possible types of correlation (e.g., intensity of seismic events, spatial and temporal distributions), the kinematic correlation of faults and focal mechanisms and its relationships with stochastic stress fields have been investigated in Kagan and Knopoff (1985a, b) and Kagan (1990, 1991, 1992a, b). The characteristics of kinematic correlations expected in active faults under homogeneous stress fields, however, have not been detailed, even though the presence of homogeneous stress fields is frequently inferred in structural geology and seismological analyses. This paper presents a GIS customization based on ArcView 3 that calculates the kinematic correlations of fault and focal mechanism populations, simulates random fault orientations and determines expected slip vectors under a homogeneous stress field. Equivalent and faster Fortran 90/95 versions of these tools are also provided; these are useful when analysing large datasets.

2. Kinematic correlation of faults and focal mechanisms

The correlation in orientation between two geometrically equivalent structures, i.e., two lines, planes or stress tensors, can be measured through algebraic or geometric indices. In the first case, a single scalar index is produced, while, in the second, a rotation angle and its associated rotation pole measure the disorientation between two structures. The statistical distribution of correlation in a population can be derived following two different methods. A *reference* orientation (e.g., the regional orientation of the kinematic axes, the orientation of the fault segment on which the main shock occurred) can be compared with all observations in the sample; this is termed a *central* statistic. In a second approach, no *a priori* reference value is considered and the resulting statistics are *pairwise*, i.e., the correlation is calculated for each possible pair of observations that constitute the sample. In this case, the influence of spatial separation and time lag on the correlation statistics can be investigated.

The kinematic correlation in faults and focal mechanisms can be investigated with the same analytical tools when we convert the fault datum into a focal mechanism. With this operation, we loose the information related to the rotational component of the finite strain tensor of the fault, maintaining the component related to the symmetric part of the strain tensor, that can be expressed through the familiar concepts of P and T axes (Cladouhos and Allmendinger, 1993).

☆ Code available from server at 〈http://www.iamg.org/CGEditor/index.htm〉.

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2.1. Scalar measures

One simple correlation index for line or plane structures is the scalar product of unit vectors parallel to linear data or normal to planar data. However, structures such as focal mechanisms, faults with slickenlines, or stress tensors require more complex procedures. For instance, Michael (1987) defined the *closeness* (i.e., correlation) between stress tensors as the scalar (double dot) product of two symmetric tensors scaled by the product of their square-rooted magnitudes:

$$\frac{\sum_{i=1}^{3}\sum_{j=1}^{3} M_{ij}N_{ij}}{(\sum_{i=1}^{3}\sum_{j=1}^{3} M_{ij})^{1/2}(\sum_{i=1}^{3}\sum_{j=1}^{3} N_{ij})^{1/2}} \tag{1}$$

This index varies from -1 for two opposite tensors to 1 for equal tensors.

Kagan and Knopoff (1985a, b) and Kagan (1992a) calculated the correlation between focal mechanisms (converted into seismic moment tensors) with their coherence index:

$$\text{coherence} = \sum_{i=1}^{3}\sum_{j=1}^{3} M_{ij}N_{ij}/(-I_2) \tag{2}$$

This is the scalar product of the normalised (symmetric) seismic tensors, scaled by the opposite of the second invariant of the normalised seismic tensor (eq. on p. 639 of Kagan and Knopoff, 1985b):

$$I_2 = L_1L_2 + L_2L_3 + L_1L_3 \tag{3}$$

where L_1, L_2 and L_3 are the eigenvalues of the tensor. As these are normalised seismic tensors, their eigenvalues are $L_1=1$, $L_2=0$ and $L_3=-1$, so $I_2=-1$. The value of the Kagan and Knopoff coherence index is simply twice Michael's closeness index. Since Kagan and co-workers extensively investigated focal mechanisms and fault correlation, we use their approach to make it easier to compare results. Equations for calculating the seismic tensor are listed in Appendix A.

2.2. Rotational measures

A rotational index of the correlation between two structural measures is the minimum angle required to rotate one measure to the same orientation of the second one. This also provides the orientation of the rotation pole, which could be useful for structural analyses. For lines and planes, this measure is simply obtained from the vector and scalar products of vectors, similar to the operations for the scalar measure of correlation. Rotations of focal mechanisms and fault planes with slickenlines are more complex and can be derived through rotation matrices or quaternion notations (Kagan, 1991; Kuipers, 2002). As they have orthorhombic symmetry, four possible solutions are produced, with the minimum angle solution between 0° and 120° (Kagan, 1991). In some cases, the four possible solutions may differ only slightly in rotation angle but significantly in orientation. This ambiguity can be dealt with by checking their consistency with the rotation poles of associated vector-type structural data (e.g., single *P*, *T* and *B* axes, fault planes, slickenlines) or by simply picking the solution with the minimum rotation angle.

2.2.1. Rotation matrices

A rotation operator in R^3 may be represented by an orthogonal matrix R (i.e., $RR^t=R^tR=I$, where I is the identity matrix) whose determinant is equal to 1 (Kuipers, 2002). The rotation matrix can be calculated given a rotation pole and angle using the formulas in Appendix B. If we consider a rotation expressed by a change of orthonormal bases, i.e., from $B_1(\mathbf{x_1}, \mathbf{y_1}, \mathbf{z_1})$ to $B_2(\mathbf{x_2}, \mathbf{y_2}, \mathbf{z_2})$, the rotation matrix is derived from the scalar products of the vectors $\mathbf{x}, \mathbf{y}, \mathbf{z}$ representing the two bases (Kuipers, 2002, Eq. 7.8):

$$R = \begin{bmatrix} x_1\cdot x_2 & x_1\cdot y_2 & x_1\cdot z_2 \\ y_1\cdot x_2 & y_1\cdot y_2 & y_1\cdot z_2 \\ z_1\cdot x_2 & z_1\cdot y_2 & z_1\cdot z_2 \end{bmatrix} \tag{4}$$

A vector $\boldsymbol{v}_1$ can be rotated using the rotation matrix R with the formula (Kuipers, 2002, p. 113):

$$\boldsymbol{v}_2 = R\boldsymbol{v}_1 \tag{5}$$

while to rotate a tensor T_1, a "sandwich" operation is needed (Kagan and Knopoff, 1985a, p. 433):

$$T_2 = R(T_1R^t) \tag{6}$$

where *Rt* is the transpose of *R*.

2.2.2. Quaternions

An alternative way to determine the rotations of objects is via quaternions. Quaternions are hyper-complex numbers of rank 4 invented by Hamilton in 1843 (Kuipers, 2002). A quaternion $\mathbb{q}=q_0+q_1i+q_2j+q_3k$ is the sum of a real number q_0 and of three real numbers q_1, q_2, q_3 multiplied by the imaginary components i, j and k, respectively, where $i^2=j^2=k^2=ijk=-1$. The *norm* of a quaternion is equal to:

$$\sqrt{q_0^2+q_1^2+q_2^2+q_3^2} \tag{7}$$

so that a normalised quaternion has a unit *norm*. A pure quaternion has the real component q_0 equal to zero. The conjugate of a quaternion $\mathbb{q}$ $[q_0, q_1, q_2, q_3]$ is $\mathbb{q}^*$ $[q_0, -q_1, -q_2, -q_3]$. For a normalised quaternion $\mathbb{q}$, the inverse $\mathbb{q}^{-1}$ (i.e., $\mathbb{q}^{-1}\mathbb{q}=\mathbb{q}\mathbb{q}^{-1}=1$) is simply $\mathbb{q}^*$.

The quaternion product $\mathbb{r}=\mathbb{p}\mathbb{q}$ in matrix notation is (Eq. 5.3 in Kuipers, 2002):

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} p_0 & -p_1 & -p_2 & -p_3 \\ p_1 & p_0 & -p_3 & p_2 \\ p_2 & p_3 & p_0 & -p_1 \\ p_3 & -p_2 & p_1 & p_0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{8}$$

The interpretation of normalised quaternions (of rank 4) as rotational operators is the extension to R^3 of the property of a normalised ordinary complex number (rank 2) to represent rotations in R^2. A normalised quaternion $\mathbb{q}=q_0+q_1i+q_2j+q_3k$ represents a rotation operator whose rotation pole is (Kagan, 1991, Kuipers, 2002):

$$\boldsymbol{n} = q_1\mathbf{i}+q_2\mathbf{j}+q_3\mathbf{k} \quad (\mathbf{i}, \mathbf{j} \text{ and } \mathbf{k} \text{: unit vectors of an orthonormal basis}) \tag{9}$$

with a rotation angle Φ:

$$\boldsymbol{\Phi} = 2\text{acos}(q_0) \quad (-180^\circ \le \boldsymbol{\Phi} \le 180^\circ, \text{ positive following the righthand}-\text{ rule}) \tag{10}$$

Rotation sequences are equivalent to quaternion multiplication: for example, two successive rotations with equivalent normalised quaternions $\mathbb{q}_1$ and $\mathbb{q}_2$ can be composed into a single rotation through quaternion multiplication:

$$\mathbb{q}_3 = \mathbb{q}_2\,\mathbb{q}_1 \tag{11}$$

A vector $\boldsymbol{v}_1=x_1\mathbf{i}+y_1\mathbf{j}+z_1\mathbf{k}$ can be rotated via a "sandwich" operation to a new orientation $\boldsymbol{v}_2=x_2\mathbf{i}+y_2\mathbf{j}+z_2\mathbf{k}$. To do this, vectors are converted into pure quaternions, i.e., $\mathbb{V}_1=0+x_1i+y_1j+z_1k$ and $\mathbb{V}_2=0+x_2i+y_2j+z_2k$; the following formula can then be used:

$$\mathbb{V}_2 = \mathbb{q}^*\,(\mathbb{V}_1\mathbb{q}) \tag{12}$$

Since a focal mechanism has three degrees of freedom, it can be represented by a normalised quaternion; this quaternion is the one equivalent to a rotation of the *T*, *P* and *B* axes from