International Review of Economics and Finance 43 (2016) 72–87

Contents lists available at ScienceDirect

International Review of Economics and Finance

journal homepage: www.elsevier.com/locate/iref

Systematic risk and volatility skew

Shyh-Weir Tzang [a,1], Chou-Wen Wang [b], Min-Teh Yu [c,*]

[a] Department of Finance, Asia University, Taichung 41354, Taiwan
[b] Financial Management Department, National Kaohsiung First University of Science and Technology, Kaohsiung 824, Taiwan
[c] National Chiao Tung University, Hsinchu 30010, Taiwan

ARTICLE INFO

Available online 23 October 2015

JEL classification:
G13
G14

Keywords:
Beta smile
CAPM
GARCH
Systematic risk proportion
Volatility skew

ABSTRACT

The impact of systematic risk on volatility skew is assessed in a CAPM–GARCH framework under which the relationship between asset price and market index adheres to the CAPM with each residual following an asymmetric GARCH process. From numerical analysis, we demonstrate that (1) the relation between beta and implied volatilities presents a beta smile; (2) beta can determine the shape of implied volatility curve, but systematic risk proportion (SRP) cannot; and (3) the degree of negative skewness and positive kurtosis is proportional to the SRP; however, a higher SRP does not always lead to a higher level of implied volatility.

1. Introduction

Over the past three decades, under the framework of Black and Scholes (1973), the option pricing model has been widely used for pricing derivatives contracts. However, the return process, following a geometric Brownian motion, deviates from the facts observed in practice. Empirical work has uncovered some intriguing features of asset returns, such as volatility clustering, leverage effect, the negative skewness, and positive excess kurtosis and the volatility smile/skew phenomenon. This motivates our study to model the dynamics of asset returns by using the discrete-time generalized autoregressive conditional heteroskedasticity (GARCH).

Moreover, empirical evidence suggests that (1) the Black–Scholes implied volatility is higher than the realized volatility; (2) the risk-neutral negative skewness is more pronounced than that in the physical distribution; and (3) the index options have a steeper slope of implied volatility curve than that of the individual equity options (e.g., Jackwerth, 2000; Dennis and Mayhew, 2002; Bakshi, Kapadia and Madan, 2003). Collectively, these features indicate structural differences between the risk-neutral and physical distributions. Because the two distributions are linked by the risk premium mainly characterized by systematic risk, the role of capital asset pricing model (CAPM) has drawn attention from academics in option pricing.

Dennis and Mayhew (2002) find that stocks with larger betas tend to have more negative skewness in risk-neutral density implied by stock option prices, thus indicating the importance of market risk in pricing individual stock options. Bakshi et al., (2003) develop a theoretical relationship between the implied volatility and the risk-neutral skewness and kurtosis, and empirically demonstrate that the differential pricing of individual stock options and index options is indeed related to their differences in the risk-neutral skewness and kurtosis. Chernov (2003) introduces systematic and idiosyncratic risk into stochastic volatility processes to estimate the pricing kernel which involves a highly nonlinear function of market portfolio returns. Branger and Schlag

* Corresponding author at: 1001 University Road, Hsinchu 30010, Taiwan. Tel.: +886 3 5 731883; fax: +886 3 5733260.
E-mail addresses: swtzang@asia.edu.tw (S.-W. Tzang), chouwen1@ccms.nkfust.edu.tw (C.-W. Wang), mtyu@nctu.edu.tw (M.-T. Yu).
[1] The author gratefully acknowledges the support from the Asia University (100-A-02).

http://dx.doi.org/10.1016/j.iref.2015.10.032
1059-0560/

(2004) decompose the diffusion and the jumps into common and idiosyncratic components in a jump-diffusion setup. Using a discrete-time GARCH option pricing, Duan and Wei (2005) express the asset risk premium as a function of systematic risk by assuming that the stochastic discount factor is a one-factor linear structure. Duan and Wei (2009), demonstrating that the price structure of individual equity options depends on the proportion of systematic variance in the total variance, further explain that the systematic risk is the driver for the behavior of the implied volatility, the risk-neutral skewness and kurtosis.

Inspired by Duan and Wei (2005, 2009) and the CAPM, this paper aims to assess the impact of systematic risk on implied volatility and risk-neutral skewness and kurtosis, by assuming that the process of stock price includes beta-a proxy of systematic risk-multiplied by the excess return of a market index in the mean equation of the stock return. This paper also assigns the normal distribution and GARCH model to each residual of the stock price and the market index, respectively. When the residuals are normal disturbances, a closed-form solution of the CAPM–Normal model is provided. However, given a fixed total (unconditional) variance, the beta or the systematic risk proportion (SRP), does not have any impact on the option values; a result not supported by the empirical findings of Duan and Wei (2009).

The main contribution of the paper is threefold. First, to incorporate the volatility clustering and the leverage effect, the asymmetric GARCH process of Heston and Nandi (2000) is assigned to each residual of stock price and market index in the CAPM–GARCH option pricing model. Based upon the Fourier transform approach of Carr and Madan (1999), this paper derives the closed-form solution under the CAPM–GARCH model. The risk-neutral skewness and kurtosis are also obtained by using the characteristic function of log stock return.

Second, within the CAPM–GARCH setup, the relation between beta and implied volatility exhibits a beta smile; a higher absolute value of beta which, in turn, leads to a higher amount of SRP that consistently results in a higher level of implied volatility and a steeper slope of the curve. Moreover, the level of beta can determine the shape of implied volatility curve, but SRP cannot. As beta increases gradually from negative to positive, the shape of implied volatility changes from a positive slope of volatility skew to a volatility smile, and eventually turning into a negative slope of volatility skew. It provides a theoretical explanation for empirical evidence that, in most cases, implied volatility curves are negative slope of volatility skew or volatility smile because the betas of stocks are almost non-negative.

The third contribution of this paper is demonstrating that the degree of the negative skewness and positive kurtosis is directly proportional to the SRP. Bakshi and Kapadia (2003) prove that the level and slope of the implied volatility curve are related to the risk-neutral skewness and kurtosis. McIntyre and Jackson (2009) try to use skewness and kurtosis of the risk-neutral distributions as predictors for changes in market direction. Duan and Wei (2009) further propose that the SRP affects the risk-neutral skewness, kurtosis and the level and slope of the implied volatility curve. Therefore, this finding is in agreement with their empirical results from a theoretical perspective. In terms of time to maturity, the longer the maturity is, the lower the degree of the negative skewness and the positive kurtosis. The SRP actually has an impact on the implied volatility; however, different from the empirical finding of Duan and Wei (2009), a higher amount of SRP does not always lead to a higher level of implied volatility, especially for the out-of-the-money options.

The remainder of this paper is organized as follows. Section 2 describes the setting of systematic risk in option pricing and stochastic processes of stock price and market index. Section 3 provides a CAPM–GARCH model in which the volatilities follow an asymmetric GARCH-in-the-mean process. The closed-form option pricing formula, together with the risk-neutral skewness and kurtosis, are derived. Section 4 provides numerical analysis to demonstrate the impact of systematic risk on implied volatility and risk-neutral skewness and kurtosis. Section 5 concludes.

2. The setting of systematic risk in option pricing

To account for the impact of systematic risk on stock returns, the dynamics of stock price S and market index M, defined in a filtered probability space$(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{t\in[0,T]})$, are specified-in the CAPM representation-as follows:

$$\ln\left(\frac{S(t)}{S(t-1)}\right) = \mu_S(t) - \frac{1}{2}h_S(t) + \beta_S\left(\ln\left(\frac{M(t)}{M(t-1)}\right) - r\right) + \sqrt{h_S(t)}z_S(t), \tag{1}$$

$$\ln\left(\frac{M(t)}{M(t-1)}\right) = \mu_M(t) - \frac{1}{2}h_M(t) + \sqrt{h_M(t)}z_M(t), \tag{2}$$

where

$$\mathrm{E}(z_S(t)z_M(t)|\mathcal{F}_{t-1}) = 0; \tag{3}$$

$\mu_S(t)$ and $\mu_M(t)$ are drift terms; β_S denotes the beta, a measure of systematic risk for the stock; r is the risk-free interest rate; $h_S(t)$ ($h_M(t)$) denotes the $\mathcal{F}_{t-1}$ conditional variance of the residual of the stock price (market index); $z_S(t)$ and $z_M(t)$, conditional on $\mathcal{F}_{t-1}$, are independent standard normal disturbances for the stock price and the market index, respectively. According to Eq. (1), it is obvious that the conditional variance of stock return depends not only on its own conditional variance but also on the conditional variance of the market index.