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# Forecasting Value-at-Risk using block structure multivariate stochastic volatility models[☆]

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ABSTRACT

Most multivariate variance or volatility models suffer from a common problem, the "curse of dimensionality". For this reason, most are fitted under strong parametric restrictions that reduce the interpretation and flexibility of the models. Recently, the literature has focused on multivariate models with milder restrictions, whose purpose is to combine the need for interpretability and efficiency faced by model users with the computational problems that may emerge when the number of assets can be very large. A contribution to this strand of the literature including a block-type parameterization for multivariate stochastic volatility models is provided. The empirical analysis on stock returns on the US market shows that 1% and 5% Value-at-Risk thresholds based on one-step-ahead forecasts of covariances by the new specification are satisfactory for the period including the Global Financial Crisis.


## 1. Introduction

Classical portfolio allocation and management strategies are based on the assumption that risky return series are characterized by time-invariant moments. However, the econometric literature of the last few decades has demonstrated the existence of dynamic behaviour in the variances of financial returns series. The introduction of such empirical evidence may constitute an additional source of performance for portfolio managers, as evidenced by Fleming, Kirby, and Ostdiek (2001), or may be relevant for improving the market risk measurement and monitoring activities (see, for example, Hull and White, 1987, Lehar, Scheicher, and Schittenkopf, 2002, and Hammoudeh and McAleer, 2015). Two families of models have emerged in the literature, namely GARCH-type specifications (see Engle, 2002), and Stochastic Volatility models (see Taylor, 1986 and Andersen, 1994).

However, portfolio management strategies often involve a large number of assets requiring the use of multivariate specifications. Among the possible alternative models, we cite the contributions of Bollerslev (1990), Engle and Kroner

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(1995), Ling and McAleer (2003), Asai and McAleer (2006, 2009a,b), and the surveys in McAleer (2005), Bauwens, Laurent, and Rombouts (2006), Asai, McAleer, and Yu (2006), and Chib, Omori, and Asai (2009). Most models, if not all, suffer from a common problem, the well-known "curse of dimensionality", whereby models become empirically infeasible if fitted to a number of series of moderate size (in some cases, the models may become computationally intractable for even 5 or 6 assets). In order to match the need of introducing time-varying variances with practical computational problems, several restricted models are generally used: the diagonal VECH specifications suggested by Bollerslev, Engle, and Woodridge (1988), the scalar VECH and BEKK models proposed by Ding and Engle (2001), the CCC model of Bollerslev (1990), the dynamic conditional correlation model of Engle and Sheppard (2001) and Engle (2002), and the varying conditional correlation model of Tse and Tsui (2002).

The introduction of strong restrictions can affect the interpretation and flexibility of the models, with a possibly impact on the purportedly improved performance they may provide and/or the appropriateness of the analysis based on their results. For example, the scalar BEKK model can reduce the number of parameters by assuming all the elements of the cross-products of the vector of past residuals have the same parameter, and the assumption can be tested by applying the asymptotic results of Hafner and Preminger (2009) if we can avoid the problem of dimensionality.

Recently, the literature has focused on multivariate models with milder restrictions, whose purpose is to combine the need for interpretability and efficiency faced by model users with the computational problems that may emerge when the number of assets is quite large. Among the contributions in this direction, we follow the approach of Billio, Caporin, and Gobbo (2006). They propose specifying the parameter matrices of a general multivariate correlation model in a block form, where the blocks are associated with assets sharing some common feature, such as the economic sector. Our purpose is to adopt this block-type parameterization and adapt it to multivariate stochastic volatility models.

In general terms, multivariate stochastic volatility (MSV) models have a parameter number of order $O(M^2)$, where $M$ is the number of assets. With the introduction of block parameter matrices, we may control the number of parameters and obtain a model specification which is feasible, even for a very large number of assets. Furthermore, as in the contribution of Billio et al. (2006), the models we propose follow the spirit of sectoral-based asset allocation strategies since they will presume the existence of common dynamic behaviour within assets or financial instruments belonging to the same economic sector. This assumption is not as strong as postulating the existence of a unique factor driving all the variances and covariances, since the financial theory may suggest the existence of sector-specific risk factors (sectoral asset allocation is often followed by portfolio managers and characterized by a number of managed financial instruments).

As distinct from an extremely restricted model, we also recover part of the spillover effect between variances, which allows monitoring of the interdependence between groups of assets, an additional element that may be relevant. Within our modelling approach, the coefficients may be interpreted as sectoral specific, while the assets will be in any case characterized by a specific long term variance through the introduction of unrestricted constants in the variance equations.

For the purpose of explaining our approach, we consider a multi-component MSV model allowing leverage effects and heavy-tailed unconditional distributions, which is a multivariate extension of Chernov, Gallant, Ghysels, and Tauchen (2003), although our approach is applicable to the factor model of Pitt and Shephard (1999) and Chib, Nardari, and Shephard (2006) and the dynamic correlation model of Asai and McAleer (2009b).

Clearly, the restrictions proposed may not necessarily be accepted by the data, as more 'complete' models will, in general, provide better results. We will show that the introduction of such restrictions provides limited losses, while yielding a significant improvement over the more restricted specifications. In addition, the restrictions make the model computationally feasible, when 'complete' models are, in general, computationally infeasible in cases where the number of assets increases. We also evaluate and compare the out-of-sample forecast of alternative models. It is worth emphasizing a further point: multivariate conditional volatility models have been used widely in the literature, but the most well known, such as DCC, has no known regularity or statistical properties (see, for example, Caporin & McAleer, 2012, 2013). On the other hand, multivariate stochastic volatility models have well established statistical properties so that, on statistical grounds, the latter are to be preferred.

The plan of the remainder of the paper is as follows. Section 2 presents the multi-component MSV models, and discusses the differences between the MSV model and the factor specifications. Section 3 introduces the block-structure modelling approach, and addresses some estimation issues. Section 4 presents an empirical example regarding the out-of-sample forecasts, based on US stock market data for selected firms. Section 5 gives some concluding comments.

## 2. Multi-component MSV model

The block-structure model, which we will present in the next section, can be considered as a restricted specification of a general MSV model. In fact we will show how the modelling approach consists in defining a set of parametric restrictions that makes the model feasible, but without losing the interpretation of the coefficients.

We define a MSV model which contains multi-components and accommodates leverage effects. Let $R_t$ be the $M$-dimensional vector of asset returns, and define $y_t = R_t - \mu_t$, where $\mu_t = E(R_t \Im_{t-1})$ is the $M$-dimensional vector of conditional means and $\Im_t$ is the information set up to $t$. Then, the mean equation of the basic MSV model is defined by

$$y_t = D_t \varepsilon_t, \tag{1}$$

$$D_t = \mathrm{diag}\{\exp(0.5 h_t)\}, \tag{2}$$

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