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Wavelet and rescaled range approach for the Hurst coefficient for short and long time series

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Abstract

The calculation of Hurst coefficient (H) by different techniques is sensitive to the length of the profile and noise. Synthetic fractional Brownian motions with different values of H have been generated and the effectiveness of the techniques has been tested on these time series. H values are calculated by wavelet transform (WT), power spectrum (PS), roughness length (RL), semi-variogram (SV), and rescaled range (R/S) methods. On the basis of the error estimates two methods: R/S analysis and WT are suggested for calculation of H for short/long datasets. Further, WT method is applied to geophysical data of the Bay of Bengal. The gravity, magnetic and bathymetry data indicate the self-affine nature with $H = 0.8$, 0.8 and 0.9, respectively.

Keywords: Self-affine; Hurst coefficient; Rescaled range; Power spectrum; Wavelet transform

1. Introduction

Many natural phenomenon such as earthquake distribution, biological system, geological structures and geophysical properties, etc. are described as self-affine fractals (Korvin, 1992; De Santis et al., 1997; Turcotte, 1997; Dimri, 2000; Bansal and Dimri, 2005a). Self-affine fractals are generalization of fractional Brownian motion (FBM) and fractional Guassian noises (Turcotte, 1997; Malamud and Turcotte, 1999). The FBM is characterized by Hurst coefficient (H), which describes the scaling nature of the motion and its value varies from 0 to 1. The Hurst coefficient can be calculated by various methods (power spectrum (PS), roughness length (RL), semi-variogram (SV), wavelet transform (WT) and rescaled range (R/S) analysis).

In practical situations, the data is of limited duration with gaps or noises and non-stationary (Yiou et al., 2000; Bansal and Dimri, 2001) and calculation of the Hurst coefficient is not reliable for the short or noisy time series (Arneodo et al., 1995; Simonsen et al., 1998; Katsev and Heureux, 2003). Katsev and Heureux (2003) pointed out the effect of irregularities in the time series using window length method and analyzed the sensitivity of H calculation to the length of the series. Here, we have compared different techniques for calculation of H for different lengths of the FBM. The aim of this paper is to find a suitable method for calculating the

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Hurst coefficient from short and long time series. The paper is organized in six sections. In Section 2, the successive random addition (SRA) method for generating the FBM is presented. For calculation of H values following methods are generally used: (a) WT method; (b) PS; (c) RL; (d) SV; (e) R/S analysis and presented in Section 3. The applications of these methods to the synthetic FBM are presented in Section 4. In Section 5, application to the gravity, magnetic and bathymetry data of the Bay of Bengal is presented. In Section 6, conclusion of the study is presented.

2. SRA method for generating FBM

In the SRA method, the FBMs are generated by assuming Gaussian probability distribution with zero mean and unit variance (Turcotte, 1997; Jones et al., 1996). First three points are selected at abscissa 0, $\frac{1}{2}$ and 1 and the values at these points are calculated using Gaussian distribution. Then, the values at midpoints of above points are linearly interpolated to give five values. Five random values are generated with zero mean and variance $V = (1/n)^{2H}$ (where n is number of values generated by interpolation) and added to the above five values. Again these five values are interpolated at mid points and the same process is repeated. In this way, one can generate the required length of FBMs with different values of H.

3. Different methods of calculating the Hurst coefficients

3.1. WT method

Application of WT has been recently suggested for calculating the Hurst coefficients (Simonsen et al., 1998; Arneodo et al., 1995; Jones et al., 1996). WT has provided an opportunity to analyze

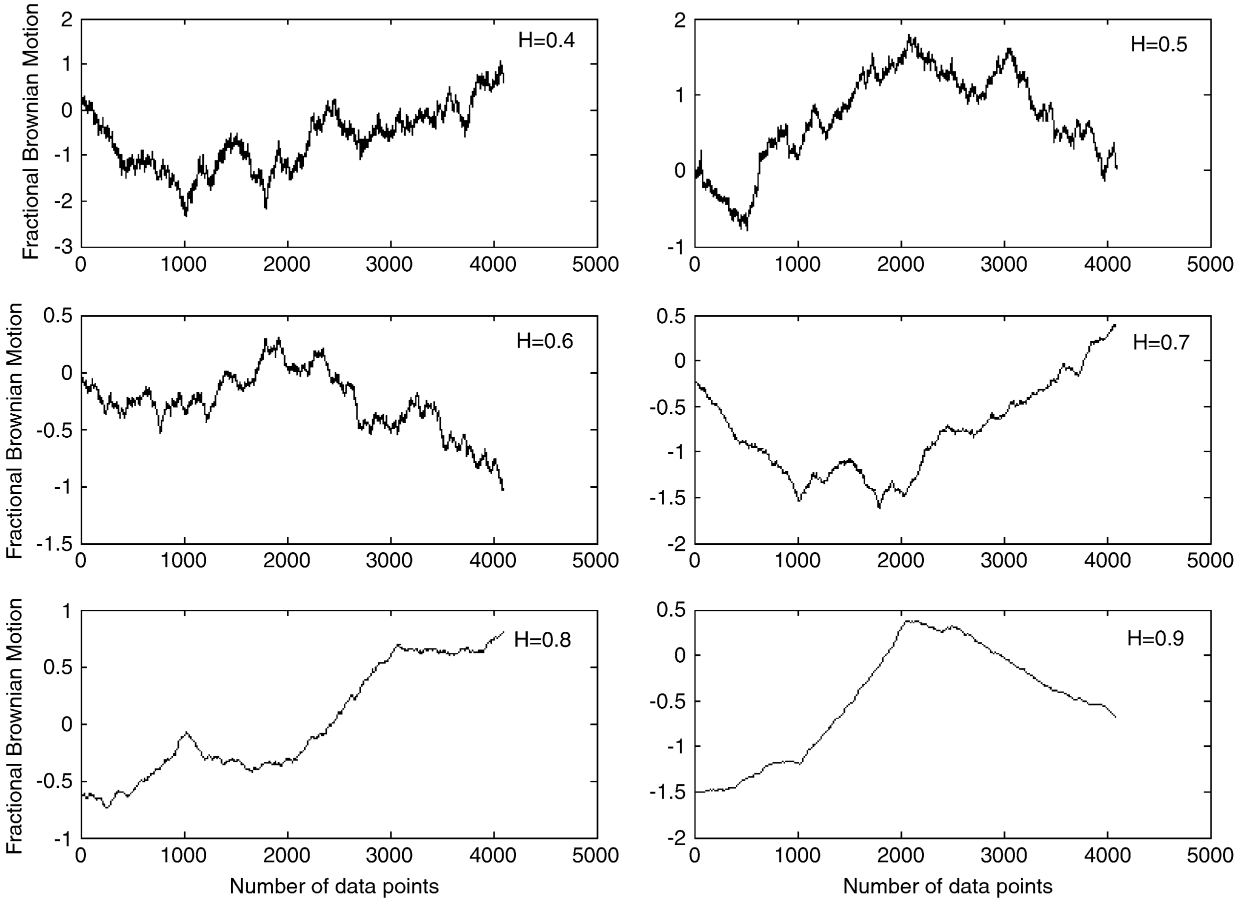

Fig. 1. Fractional Brownian motion generated for H values variation from 0.4 to 0.9.