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International trade and hedging under joint price and exchange rate uncertainty

Kit Pong Wong *

School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong

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ABSTRACT

This paper examines the behavior of a competitive exporting firm under joint price and exchange rate uncertainty. We show that the firm's optimal production and hedging decisions depend crucially on the degree of forward market incompleteness, and on the correlation structure of the price and exchange rate risk. The separation theorem holds if there are complete forward markets for hedging purposes. Should the forward markets be incomplete, the firm employs operational hedging by adjusting its output so as to better cope with the residual risk that is unhedgeable by simply trading the existing forward contracts. In the case that the price risk cannot be directly managed by financial hedging, we construct a reasonable example in which the firm optimally produces more, not less, than the benchmark output level under perfect hedging. Our results thus offer new insights into the interaction between financial and operational hedging in the context of multiple sources of uncertainty.

1. Introduction

The literature on the behavior of a competitive exporting firm under exchange rate uncertainty is abundant. One important strand of this literature examines the firm's optimal production and hedging decisions when a currency forward market exists (see, e.g., Adam-Müller, 1997; Broll & Eckwert, 1998, 2000; Broll & Zilcha, 1992; Katz & Paroush, 1979; Machnes, 1992; Wong, 2002, 2006, 2007; to name just a few). Two notable results emanate. First, the separation theorem states that the firm's optimal production decision depends neither on the risk attitude of the firm nor on the incidence of the underlying exchange rate uncertainty. Second, the full-hedging theorem states that the firm should completely eliminate its exchange rate risk exposure by adopting a full-hedge should the currency forward market be unbiased.[1]

The purposes of this paper are to contribute to this literature by examining the behavior of a competitive exporting firm that faces not only exchange rate uncertainty but also output price uncertainty as in Benninga, Eldor, and Zilcha (1985), Kawai and Zilcha (1986), Viaene and Zilcha (1998), and Wong (2012). We consider three scenarios that are characterized by different degrees of forward market incompleteness. In the first scenario, the firm has access to both commodity and currency forward markets, which serves as a benchmark wherein the firm can conduct perfect hedging against its joint price and exchange rate risk

* Tel.: +852 2859 1044; fax: +852 2548 1152.
E-mail address: kpwong@econ.hku.hk.

[1] The full-hedging theorem is analogous to a well-known result in the insurance literature that a risk-averse individual fully insures at an actuarially fair price (see Mossin, 1968).

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exposure. We show that the separation theorem holds. The full-hedging theorem, however, holds if the random spot exchange rate and the random output price are uncorrelated.

In the remaining two scenarios, we assume that either the commodity forward market or the currency forward market exists for hedging purposes. When the currency forward market is missing, we show that the firm optimally opts for an under-hedge (over-hedge) in the commodity forward market if the random spot exchange rate and the random output price are negatively (positively) correlated. Should these two random variables be independent, the full-hedging theorem holds. The firm optimally produces less and shrinks its exports to the foreign country as compared to the benchmark case of perfect hedging. In response to such forward market incompleteness, the firm employs operational hedging by cutting down its output so as to limit the residual risk that is unhedgeable by simply trading the commodity forward contracts.

When the commodity forward market is missing, we show that the full-hedging theorem may or may not hold even when the random spot exchange rate and the random output price are independent. In this case, if the firm's preferences satisfy prudence in the sense of Kimball (1990, 1993), a precautionary motive arises that induces the firm to opt for a long position in the currency forward market, thereby rendering the optimality of an under-hedge. While the firm finds it optimal to produce less than the benchmark level under perfect hedging if the random spot exchange rate and the random output price are either uncorrelated or negatively correlated, we construct a reasonable example in which the firm's optimal output level can in fact be larger should the correlation between the two random variables be sufficiently positive. Our results thus suggest that financial hedging with forwards and operational hedging with output interact in a rather complicated manner that demand a close scrutiny.

The rest of the paper is organized as follows. Section 2 develops the model of a competitive exporting firm facing joint price and exchange rate uncertainty. Section 3 derives the firm's optimal production and hedging decisions when both commodity and currency forward markets are present. Section 4 examines the firm's optimal production and hedging decisions when the currency forward market is missing. Section 5 goes on to examine the firm's optimal production and hedging decisions when the commodity forward market is missing. The final section concludes.

2. The model

Consider a competitive exporting firm that operates for one period with two dates, 0 and 1. The firm possesses a von Neumann-Morgenstern utility function, $U(\Pi)$, defined over its home currency profit at date 1, Π. The firm is risk averse so that $U'(\Pi)>0$ and $U''(\Pi)<0$ for all $\Pi>0$.

To begin, the firm produces a single commodity in the home country. The firm's production technology exhibits decreasing returns to scale, which gives rise to a deterministic cost function, $C(Q)$, compounded to date 1, where $Q\geq 0$ is the output level chosen by the firm at date 0, $C(0)=C'(0)=0$, and $C'(Q)>0$ and $C''(Q)>0$ for all $Q>0$. The firm sells its entire output, Q, in a foreign country at date 1 at the then prevailing price, $\tilde{P}$, that is not known ex ante and is denominated in the foreign currency per unit of the commodity.[2] The spot exchange rate, $\tilde{S}$, which is expressed in units of the home currency per unit of the foreign currency at date 1, is also not known ex ante. The two random variables, $\tilde{S}$ and $\tilde{P}$, are possibly correlated.

To hedge against its exposure to the joint price and exchange rate uncertainty, the firm can trade forward contracts at date 0. We consider three scenarios that are characterized by different degrees of forward market incompleteness. In the first scenario, the firm has access to complete forward markets. Specifically, the firm can sell (purchase if negative) X units of the commodity forward at the forward price, P^{f}, and Y units of the foreign currency forward at the forward exchange rate, S^{f}, where P^{f} is denominated in the foreign currency, S^{f} is expressed in units of the home currency per unit of the foreign currency, and both P^{f} and S^{f} are predetermined at date 0. The firm's random profit at date 1, denominated in the home currency, is therefore given by

$$\tilde{\Pi}=\tilde{S}\tilde{P}Q+\tilde{S}\left(P^{\mathrm{f}}-\tilde{P}\right)X+\left(S^{\mathrm{f}}-\tilde{S}\right)Y-C(Q), \tag{1}$$

where $\tilde{S}\left(P^{f}-\tilde{P}\right)X$ is the gain (loss if negative) from the forward position, X, in the commodity forward market, and $\left(S^{\mathrm{f}}-\tilde{S}\right)Y$ is the gain (loss if negative) from the forward position, Y, in the currency forward market. In the remaining two scenarios, forward markets are incomplete in that the firm has access to either the commodity forward market only so that $Y\equiv 0$, or the currency forward market only so that $X\equiv 0$.

The forward position, X, in the commodity forward market is said to be an under-hedge, a full-hedge, or an over-hedge, depending on whether X is smaller than, equal to, or greater than the output level, Q. Likewise, the forward position, Y, in the currency forward market is said to be an under-hedge, a full-hedge, or an over-hedge, depending on whether Y is smaller than, equal to, or greater than the expected foreign currency revenue, $E\left(\tilde{P}\right)Q$, respectively, where $E(\cdot)$ is the expectation operator with respect to the joint cumulative distribution function of $\tilde{P}$ and $\tilde{S}$. If $X=Q$ and $Y=E\left(\tilde{P}\right)Q$, the firm is said to adopt a double full-hedge.

[2] Throughout the paper, random variables have a tilde (˜) while their realizations do not.