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Return distribution predictability and its implications for portfolio selection

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A B S T R A C T

The inquiries to return predictability are traditionally limited to conditional mean, while literature on portfolio selection is replete with moment-based analysis with up to the fourth moment being considered. This paper develops a distribution-based framework for both return prediction and portfolio selection. More specifically, a time-varying return distribution is modeled through quantile regressions and copulas, using quantile regressions to extract information in marginal distributions and copulas to capture dependence structure. A preference function which captures higher moments is proposed for portfolio selection. An empirical application highlights the additional information provided by the distributional approach which cannot be captured by the traditional moment-based methods.

1. Introduction

Return predictability is of profound importance in many fields of finance such as asset pricing and portfolio management, and hence has been one of the most researched areas in finance for decades. The economic method employed in a typical study on return predictability is a predictive regression which is an ordinary least squares (OLS) regression of returns on lagged predictor variables. The use of predictive regressions to describe relations between returns and predictor variables implicitly assumes that the predictor variables have homogenous effects across the distribution of returns. This is an over-stringent assumption, as this paper will show later, and often yields a incomplete and even misleading picture of variable effects on returns. Furthermore, the use of predictive regressions can only offer a conditional mean view. Return predictability should be more than the first moment.

(Cenesizoglu & Timmermann, 2008) are amongst the first ones to extend the predictability inquires from the mean to the full distribution. They study the predictability of the distribution of the S&P500 Index returns using lagged predictor variables. By employing a quantile regression framework, they find significant predictability, both in sample and out-of-sample, of the entire stock return distribution. However, they only consider a single asset return distribution, whereas the joint asset return distribution is critical in economics and finance. This is especially the case for portfolio management as almost all investment decisions involve more than one asset. It is therefore of great importance to investigate multiple assets and their joint distributions.

(Pedersen, 2010) also uses quantile regressions to examine the predictability of the distributions of the S&P500 Index and the US 5-year Treasure bonds. He proposes to model the joint asset return through the use of multivariate quantile regression, *i.e.*, a single regression model with more than one outcome variable. Although the multivariate quantile regression can yield some insights into the joint asset returns, it has disadvantages. The parameter estimation for multivariate quantile regression is challenging, especially computationally, as the estimates are often unstable and not unique. Additionally, constructing a joint distribution using multivariate quantile regression is not a straightforward task. It involves estimation of a sufficiently fine grid of multidimensional quantiles of which the computational intensity increases exponentially with the number of dimensions.

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The research in return distribution predictability only just starts. This paper aims to provide insights into this new area with a threefold purpose. First, it complements the literature on return distribution predictability by providing further empirical evidence using two broad-based indices: the Russell 1000 Index and the US Aggregate Bond Index. Compared to the S&P 500 Index and the 5-year Treasure bonds studied by (Cenesizoglu & Timmermann, 2008) and (Pedersen, 2010), these two broad-based indices are more comprehensive and unbiased barometers for the US stock and bond markets. Given their wide recognition in investment communities, predictability of these indices has academic value as well as significant economic value to investors. The empirical analysis involves regressing monthly returns of the stocks and bonds on a range of economic state variables in a quantile regression setup. A number of the economic state variables considered show significant but heterogenous effects on various parts of the return distributions. This is especially pronounced for the bond returns.

Second, this paper proposes a general and flexible framework to model the joint return distribution. The building blocks of the framework are quantile regressions and copulas. The new framework hinges on quantile regressions for marginal return distributions and a copula for dependence structure across asset returns. This quantile-copula framework is demonstrated to be convenient and flexible to model a joint distribution while, at the same time, capturing any non-Gaussian characteristics in both marginal and joint returns. It also remains tractable even when several assets are considered. Importantly, the well-developed copula theory is a more convenient vehicle for analysis than the less-developed multivariate quantile regression approach.

Last, the paper makes an initial attempt to explore implications of the return distribution predictability on portfolio management. To make a full use of the conditional return distribution modeled through the quantile-copula approach, the Omega measure proposed by (Shadwick & Keating, 2002) is modified and used as a preference function. This preference measure captures the full joint return distribution information and portfolio selection under the proposed measure is intuitively appealing and empirically implementable.

The reminder of the paper is organized as follows. Section 2 provides rationale on using quantile regressions for return prediction and introduces the quantile-copula approach for joint return modeling. Section 3 discusses a distribution-based framework for portfolio selection and proposes a preference function for the purpose. A detailed description of the stock and bond data is given in Section 4. Section 5 presents the empirical findings and Section 6 concludes. Appendix A describes the optimization algorithm used, while Appendix B provides the functional forms of the copula models considered.

2. Joint return distribution modeling

2.1. Marginal distribution by quantile regressions

In the past few decades, a great deal of research on return predictability has been dedicated to predictive regressions under Gaussian conditions. A typical predictive regression is to regress the return {r_{{t + 1}}} on a lagged predictor variable x_t,

$$r_{t+1} = \beta_0 + \beta_1 x_t + \epsilon_{t+1}, \tag{1}$$

where ${}_{t+1}$ is a return innovation which follows a normal distribution $N(0,\sigma)$. If F_t is the information available at time t and Φ is a standard normal cumulative distribution, then the implied τ-th conditional quantile of r_{t+1} by the model (1) is

$$Q_\tau(r_{t+1}|F_t) = \beta_0 + \beta_1 x_t + \sigma \Phi^{-1}(\tau) \equiv \beta_{0,\tau} + \beta_1 x_t, \tau \in (0,1),$$

where $\Phi^{-1}(\tau)$ is the τ-th quantile of a standard normal distribution. Across the distribution of r_{t+1}, the only parameter that changes with τ is the location $\beta_{0,\tau}$ which is determined solely by the mean effect β_0 and the standard error of the innovation (or conditional volatility) σ. Therefore, despite the linear regression model (1) being designed only to capture the conditional mean effect, in an ideal Gaussian world it provides a complete view of the future return.

In real life, however, a single mean curve and the associated conditional volatility are rarely adequate summaries of the relationship between returns and covariates. Stock returns are commonly observed to exhibit non-Gaussian features. By replacing the Gaussian distribution assumption with a general distribution F for the return innovation ${}_{t+1}$, even in the simplest case in which the covariate effect is constant across all quantiles, the τ-th conditional quantile of r_{t+1},

$$Q_\tau(r_{t+1}|F_t) = \beta_0 + F_\epsilon^{-1}(\tau) + \beta_1 x_t,$$

is no longer determined solely by the mean effect and conditional volatility. Instead it involves estimation of the distribution F.

Furthermore, there is no compelling theoretical reason to believe that β_1 should be constant across quantiles. If some of the slope coefficients change with the quantile τ, then this is indicative of some form of heteroscedasticity. This can occur when predictor variables not only affect the conditional mean, but are also linked to the conditional variance (as is often the case in finance applications). An example is given below. Suppose

$$r_t = \beta_0 + \beta_1 x_{t-1} + \sigma_t \in_t,$$
$$\sigma_t = \gamma_0 + \gamma_1 x_{t-1},$$