



Estimating the spot rate curve using the Nelson–Siegel model A ridge regression approach

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ABSTRACT

The Nelson–Siegel model is widely used in practice for fitting the term structure of interest rates. Due to the ease in linearizing the model, a grid search or an OLS approach using a fixed shape parameter are popular estimation procedures. The estimated grid search parameters, however, have been reported (1) to behave erratically over time, and (2) to have relatively large variances. On the other hand, parameter estimates based on a fixed shape parameter, while avoiding multicollinearity, turn out to be too smooth. We show that the Nelson–Siegel model can become heavily collinear depending on the estimated/fixed shape parameter. A simple procedure based on ridge regression can remedy the reported problems significantly.

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1. Introduction

Good estimates of the term structure of interest rates are of the utmost importance to investors and policy makers. One of the term structure estimation methods, initiated by Bliss and Fama (1987), is the smoothed bootstrap which bootstraps discrete spot rates from market data and then fits a smooth and continuous curve to the data. Although various curve fitting spline methods have been introduced (quadratic splines by McCulloch, 1971; cubic splines by McCulloch, 1975; exponential splines by Vasicek & Fong, 1982; B-splines by Shea, 1984), these methods have been criticized on the one hand for having undesirable economic properties and on the other hand for being ‘black box’ models (Seber & Wild, 2003). Nelson and Siegel (1987) and Svensson (1994, 1996) therefore suggested parametric curves that are flexible enough to describe a whole family of observed term structure shapes. These models are parsimonious, consistent with a factor interpretation of the term structure (Litterman & Scheinkman, 1991) and have both been widely used in academia and in practice. In addition to the level, slope and curvature components present in the Nelson–Siegel (NS) model, the Svensson model contains a second hump/trough term which allows for an even broader and more complicated range of term structure shapes. In this paper, we restrict ourselves to the NS model. The Svensson model shares – by definition – all the reported problems of the NS approach. Since the source of the problems, i.e., collinearity, is the same for both models, the reported estimation problems of the Svensson model may be reduced analogously.

The NS model is extensively used by central banks and monetary policy makers (Bank of International Settlements, 2005; European Central Bank, 2008). Fixed-income portfolio managers use the model to immunize their portfolios (Barrett, Gosnell, &

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Heuson, 1995; Hodges & Parekh, 2006) and recently, the NS model also regained popularity in academic research. Dullmann and Uhrig-Homburg (2000) use the NS model to describe the spot rate curves of Deutsche Mark-denominated bonds to calculate the risk structure of interest rates. Fabozzi, Martellini, and Priaulet (2005) and Diebold and Li (2006) benchmarked the NS forecasts against other models in the term structure forecasts, and found it performs well, especially for longer forecast horizons. Martellini and Meyfredi (2007) use the NS approach to calibrate the spot rate curves and estimate the value-at-risk for fixed-income portfolios. Yan, Shi, and Wu (2008) use the NS model to bootstrap riskless spot rate curve as the input for calculating the credit risk spread for the U.S. market. Finally, the NS model estimates are also used as an input for affine term structure models. Coroneo, Nyholm, and Vivada-Koleva (2011) test to which degree the NS model approximates an arbitrage-free model. They first estimate the NS model and then use the estimates to construct interest rate term structures as an input for arbitrage-free affine term structure models. They find that the parameters obtained from the NS model are not statistically different from those obtained from the 'pure' no-arbitrage affine-term structure models.

Notwithstanding its economic appeal, the NS model is highly nonlinear which causes many users to report estimation difficulties. Nelson and Siegel (1987) transformed the nonlinear estimation problem into a simple linear problem, by fixing the shape parameter that causes the nonlinearity. In order to obtain parameter estimates, they computed the ordinary least squares (OLS) estimates of a series of models conditional upon a grid of the fixed shape parameter. The estimates that, conditional upon a fixed shape parameter, maximized the R^2 were chosen. We refer to their procedure as a *grid search*. Others have suggested estimating the NS parameters simultaneously using *nonlinear optimization techniques*. Cairns and Pritchard (2001), and Vermani (2012), however, show that the estimates of the NS model are very sensitive to the starting values used in the optimization. Moreover, the time series of the estimated coefficients have been documented to be very unstable (Barrett et al., 1995; de Pooter, 2007; Diebold & Li, 2006; Fabozzi et al., 2005; Gurkaynak, Sack, & Wright, 2006) and even to generate negative long term rates, thereby clearly violating any economic intuition. Finally, the standard errors on the estimated coefficients, though seldom reported, are large.

Although these estimation problems have been recognized before, it has never led towards satisfactory solutions. Instead, it became a common practice to fix the shape parameter over the whole time series of the term structures.¹ Hurn, Lindsay, and Pavlov (2005), however, point out that the NS model is very sensitive to the choice of this shape parameter. de Pooter (2007) confirms this finding and shows that with different fixed shape parameters, the remaining parameter estimates can take extreme values. We show that fixing the shape parameter can also result in extremely smooth time series of the parameter estimates, making it a non-trivial issue. To alleviate the observed problems substantially and to estimate the shape parameter freely, we use ridge regression.

The remainder of this paper is organized as follows. In Section 2, we introduce the NS model. Section 3 presents the estimation procedures used in the literature, illustrates the multicollinearity issue which is conditional on the estimated (or fixed) shape parameter and proposes an adjusted procedure based on the ridge regression. In the subsequent section (Section 4) we present our data and their descriptive statistics. Since the ridge regression introduces a bias in order to avoid multicollinearity, we mainly evaluate the merits of the models based on their ability to extrapolate the short and long end of the term structure. The estimation results and the robustness of our ridge regression are discussed in Section 5. Finally, we conclude.

2. A first look at the Nelson–Siegel model

The Nelson and Siegel (1987) spot rate function $r(\tau)$ at time to maturity τ is specified as

$$r(\tau) = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}' \begin{bmatrix} \lambda(1 - e^{-\tau/\lambda})/\tau \\ \lambda(1 - e^{-\tau/\lambda})/\tau - e^{-\tau/\lambda} \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}' \begin{bmatrix} r_0 \\ r_1 \\ r_2 \end{bmatrix}. \quad (1)$$

In Eq. (1) r_0 , r_1 and r_2 represent the level, slope and curvature components of the spot rate curve. The role of the components becomes clear when we look at their limiting behavior with respect to the time to maturity. When the time to maturity grows to infinity, the slope and curvature component vanish and the long-term spot rate converges to a constant level of interest rate, β_0 . When the time to maturity approaches zero, only the curvature component vanishes and the spot rate converges to $(\beta_0 + \beta_1)$. The spread, $-\beta_1$, measures the slope of the term structure, whereby a negative (positive) β_1 represents an upward (downward) slope. The degree of the curvature is controlled by β_2 , the rate at which the slope and curvature component decay to zero. Finally, the location of the maximum/minimum value of the curvature component is determined by λ . Note that λ determines both the shape of the curvature component and the hump/trough of the term structure. By maximizing the curvature component in the spot rate function with respect to λ , we are able to determine the location of the hump/trough of the term structure. The curvature component in the spot rate curve reaches its maximum when $\tau > \lambda$, which is determined by simply maximizing r_2 in Eq. (1) with λ fixed. Alternatively, we can force the location of the hump/trough of the term structure to be at a given time to maturity, by fixing the shape parameter to a specific value. This also linearizes the model and hence facilitates estimation (as in Diebold & Li, 2006; Fabozzi et al., 2005).

¹ Barrett et al. (1995) and Fabozzi et al. (2005) fix this shape parameter to 3 for annualized returns. Diebold and Li (2006) choose an annualized fixed shape parameter of 1.37 to ensure the stability of parameter estimation.

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