ARTICLE IN PRESS
FINANA-00924; No of Pages 7
International Review of Financial Analysis xxx (2015) xxx–xxx
Contents lists available at ScienceDirect
International Review of Financial Analysis

A generalized probability framework to model economic agents' decisions under uncertainty

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ARTICLE INFO

Available online xxxx

Jel classification:
D81

Keywords:
Econophysics
Expected utility theory
Ellsberg paradox
Quantum mathematics

ABSTRACT

The applications of techniques from statistical (and classical) mechanics to model interesting problems in economics and finance have produced valuable results. The principal movement which has steered this research direction is known under the name of 'econophysics'. In this paper, we illustrate and advance some of the findings that have been obtained by applying the mathematical formalism of quantum mechanics to model human decision making under 'uncertainty' in behavioral economics and finance. Starting from Ellsberg's seminal article, decision making situations have been experimentally verified where the application of Kolmogorovian probability in the formulation of expected utility is problematic. Those probability measures which by necessity must situate themselves in Hilbert space (such as 'quantum probability') enable a faithful representation of experimental data. We thus provide an explanation for the effectiveness of the mathematical framework of quantum mechanics in the modeling of human decision making. We want to be explicit though that we are not claiming that decision making has microscopic quantum mechanical features.

1. Introduction

Roughly speaking, 'econophysics' concerns the application of classical (and statistical mechanical) physics theories to model the behavior of economic and financial systems. The econophysics movement has been leaded by several brilliant physicists (see, e.g., (Mantegna & Stanley, 1995), (Mantegna & Stanley, 2000), (Roehner, 2002), (McCauley, 2004), (Schinckus, 2013). This article aims to bring to the attention of econophysicists a novel emerging domain where the application of methods and techniques inspired by quantum physics has been successful in the last years. This domain, known in the scientific community as 'quantum cognition', was born as a bold proposal to solve a specific problem.[1]

The quantum cognition domain applies the mathematical formalism of quantum mechanics to model situations and processes in human cognition, decision making and language that have resisted traditional modeling techniques by means of classical structures, i.e. Boolean logical structures, Kolmogorovian probability spaces, Bayesian update of probabilities, commutative algebras, etc. (see Section 2). Therefore, the results obtained in quantum cognition have a deep impact on behavioral economics and finance. This domain has attracted in the last years the interest of high impact factor and top journals, media and popular science and funding institutions (Lambert Mogiliansky, Zamir, & Zwirn, 2009), (Aerts, 2009), (Khrennikov, 2010), (Busemeyer, Pothos, Franco, & Trueblood, 2011), (Busemeyer & Bruza, 2012), (Aerts, Broekaert, Gabora, & Sozzo, 2013a), (Aerts, Gabora, & Sozzo, 2013b), (Haven & Khrennikov, 2013), (Aerts, Sozzo, & Tapia, 2014), (Yukalov & Sornette, 2014), (Sozzo, 2014), (Sozzo, 2015). To better clarify the boundaries of quantum cognition it is worth mentioning two important aspects of it, which are as follows.

(i). The success of this quantum modeling is interpreted as due to the 'descriptive effectiveness of the mathematical apparatus of quantum theory as a formal tool to model cognitive dynamics and structures in situations where classical set-based approaches are problematical', 'without any' a priori direct or precise connection with the validity of quantum laws in the microscopic world.

(ii). There is no need, in order to guarantee the validity of the obtained results, to introduce any compelling assumption about the existence of microscopic quantum processes at the level of the human brain. Hence, quantum cognition should not be confused with 'quantum mind' or 'quantum consciousness'.
What are the possible advantages of quantum cognition in economics? In this respect, the application of normative models of decision making to the behavior of economic agents has produced a variety of sophisticated mathematical frameworks, the most important of which are 'expected utility theory' (EUT) (von Neumann & Morgenstern, 1944) and 'subjective expected utility theory' (SEUT) (Savage, 1954). The former is designed for decisions under 'risk', that is, a choice among different

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[1] See, e.g., the articles "Quantum minds: Why we think like quarks", *New Scientist* 05 September 2011, by M. Buchanan, and "Physics goes social: How behavior obeys quantum logic", *New Scientist* 11 July 2013, by A. Khrennikov and E. Haven.

http://dx.doi.org/10.1016/j.irfa.2015.12.002
1057-5219/

gambles defined on an objective probability measure, whilst the latter is designed for decisions under 'ambiguity', that is, a choice among different acts defined on a subjective probability measure. Both theories implicitly assume that 'probabilities are Kolmogorovian', that is, probabilities are assigned to events according to rules obeying the axioms of Kolmogorov. However, since the work ofAllais (1953), decision economists systematically produce empirical situations where concrete human decisions violate the axioms of EUT. Moreover, since the work of Ellsberg (1961), decision economists are also able to generate empirical situations where concrete human decisions violate the axioms of SEUT. Finally, recent work of Machina (2009) reveals that the most recognized extensions of EUT and SEUT able to cope with 'Allais' or 'Ellsberg paradoxes' are highly problematical in specific decision making situations, i.e. 'Machina paradox' (see Section 3). Inspired by our quantum cognition approach, we have recently elaborated a complete modeling of the 'Ellsberg paradox' by using the mathematical formalism of quantum mechanics (Aerts et al., 2014), (Khrennikov & Haven, 2009), (Aerts, Sozzo, & Tapia, 2012). We have also faithfully represented the data collected in an experiment we performed on a typical Ellsberg paradox situation with real decision makers (Aerts et al., 2014). In this paper we further inquire into our quantum-theoretic framework for the Ellsberg paradox, showing that our results go beyond the mere theoretical modeling and representation of a set of empirical data. We also provide sufficient arguments to claim that, not only in the Ellsberg paradox, but also in other situations affected by ambiguity, such as the 'Machina paradox', structurally there is a real need for a non-classical probability model. We would like to advance two reasons.

(i). In an Ellsberg-type decision making process, the agent's choice is actualized as a consequence of an interaction with the cognitive context, exactly like in a quantum measurement process where the outcome of the measurement is actualized as a consequence of the interaction of the measured particle with the measuring apparatus. Therefore, in cognitive entities, as well as in microscopic quantum entities, measurements do not reveal preexisting values of the observed properties but, rather, they actualize genuine potentialities. Classical Kolmogorovian probability can only formalize lack of knowledge about actualities, hence it is generally not able to cope with a decision making process. We have proven that this is possible by using a complex Hilbert space, and by representing probability measures by means of 'projection valued measures' on a complex Hilbert space (Aerts, 2009), (Sozzo, 2014), (Sozzo, 2015). A projection valued measure is essentially different from a single Kolmogorovian probability measure, since the latter is a σ–algebra valued measure, whilst the former is not.

(ii). The notion of ambiguity, as introduced in economics, is completely compatible, both at a mathematical and an intuitive level, with the representation of states of cognitive entities as vectors of a Hilbert space. Indeed, just like in standard quantum mechanics the state vector incorporates the 'quantum uncertainty' of a microscopic particle, also in an Ellsberg-type situation, the agent's subjective preference towards ambiguity is naturally formalized by representing the state of the cognitive entity under study by means of such a Hilbert space vector (this perspective is getting more and more accepted in the scientific community, including mainstream psychologists (see Wang, Solloway, Shiffrin, & Busemeyer, 2014). In this respect, it is worth mentioning that Ellsberg called 'ambiguity aversion' the 'irrational' factor inducing decision makers to deviate from SEUT. In our approach, ambiguity aversion is only one – albeit an important one – of the conceptual landscapes surrounding the decision maker's choice in a situation where ambiguity is present. This result is compatible with the experimental findings confirming Ellsberg's prediction about the human attitude towards ambiguity (Machina & Siniscalchi, 2014), but also with some recent experiments where such attitude is more controversial (Charness, Karni, & Levin, 2013).

Points (i) and (ii) provide an intuitive explanation for the identification of genuine quantum structures in the Ellsberg paradox. Those structures are typically characterized by notions such as 'contextuality', 'interference' and 'superposition', which will be discussed in more detail in Section 4.

In concluding this section, it is important to mention that our model which aims to represent human decision making in economics is a descriptive model: it describes what economic agents actually do, not what they should do, under uncertainty. However, it already contains some insights on how the construction of an axiomatic framework of what we could call 'contextual expected utility' as based on a non-classical probability can be able to cope with human ambiguity, or 'contextual risk', as we could call it. If we wanted to embed our approach into the fundamentals of microeconomics, then a natural generalization of EUT and SEUT may simply consist in requiring that economic agents maximize their contextual expected utility. An important achievement in that regard would require a representation theorem which provides for a rigorous proof of the equivalence between the existence of a preference relationship and an order inequality between utility functions embedding this type of expected utility.

Our generalization of the probability models employed in an expected utility framework has a profound impact on any economics or finance problem where this basic microeconomic framework is used as an input in its modeling objectives. Indeed, an important assumption in general equilibrium based macroeconomic models is the 'rational expectations hypothesis' which exactly rests on the expected utility hypothesis. The consistency of the models imposed by rational expectations has profound implications on the design and impact of macroeconomic policy-making (Hansen & Sargent, 2010), (Mehra & Prescott, 1985).

2. On the effectiveness of quantum mathematics in human cognition

Classical Boolean logic and Kolmogorovian (or Bayesian) probability theory have exercised a long influence on the way in which scholars formalize human behavior under uncertainty. However, empirical evidence, accumulated in the last thirty years in cognitive psychology, clearly indicates that these classical structures do not provide the most general modeling framework for human decision making.

There are three major domains of cognition where deviations from classical logical and probabilistic structures have been observed.

The first of these two domains is 'concept theory'. Cognitive scientists know that concepts exhibit 'graded', or 'fuzzy', 'typicality', e.g., humans estimate an exemplar such as *Robin* as more typical than *Stork* as a typical example of the concept *Bird*. A problem arises when one tries to formalize the typicality of the combination of two concepts in terms of the typicality of the component concepts which form that combination. One is intuitively led to think that the standard rules of classical (fuzzy set) logic and probability theory apply in such combinations. However, Osherson and Smith showed in 1981 that this intuition is not correct for concept conjunctions. Humans score the typicality of an exemplar such as *Guppy* with respect to the conjunction *Pet–Fish* as higher than the typicality of *Guppy* with respect to both *Pet* and *Fish* separately ('Guppy effect') (Osherson & Smith, 1981). One realizes at once that typicality violates standard rules of classical (fuzzy set) logic. A second set of human experiments on concept combinations was performed by James Hampton. He measured the membership weight, i.e. normalized membership estimation, of several exemplars, e.g., *Apple*, *Broccoli*, and *Almond*, with respect to pairs of concepts, e.g., *Fruits*, *Vegetables*, and their conjunction, e.g., *Fruits* and *Vegetables*, or

Please cite this article as: Haven, E., & Sozzo, S., A generalized probability framework to model economic agents' decisions under uncertainty, *International Review of Financial Analysis* (2015), http://dx.doi.org/10.1016/j.irfa.2015.12.002