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A thermodynamical view on asset pricing

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ABSTRACT

The dynamics of stock market systems was analyzed from the stand point of viscoelasticity, i.e. conservative and nonconservative (or elastic and viscous) forces. Asset values were modeled as a geometric Brownian motion by generating random Wiener processes at different volatilities and drift conditions. Specifically, the relation between the return value and the Wiener noise was investigated. Using a scattering diagram, the asset values were placed into a 'potentiality–actuality' framework, and using Euclidean distance, the market values were transformed into vectorial forms. Depending on whether the forthcoming vector is aligned or deviated from the direction of advancement of the former vector, it is possible to split the forthcoming vector into its conservative and nonconservative components. The conservative (or in-phase, or parallel) component represents the work-like term whereas the nonconservative (or out-of-phase, or vertical) component represents heat-like term providing a treatment of asset prices in thermodynamical terms. The resistances exhibited against these components, so-called the modulus, were determined in either case. It was observed that branching occurred in the values of modulus especially in the modulus of the conservative component when it was plotted with respect to the Euclidean distance of Wiener noise, i.e. Wiener length. It was also observed that interesting patterns formed when the change of modulus was plotted with respect to the value of Wiener noise. The magnitudes of work-like and heat-like terms were calculated using the mathematical expressions. The peaks of both heat-like and work-like terms reveal around the zero value of Wiener noise and at very low magnitudes of either term. The increase of both the volatility and the drift acts in the same way, and they decrease the number of low heat-like and work-like terms and increase the number of the ones with larger magnitudes. Most interestingly, the increase either in volatility or in drift decreases the heat-like term but increases the work-like term in the overall. Finally, the observation of the golden ratio in various patterns was interpreted in terms of physical resistance to flow.

1. Introduction

Financial markets are very complex systems involving too many parameters of natural, industrial, economic, political, social, and psychological origin. Therefore it is very difficult to analyze the contribution of each parameter and set up a universal equation to describe the behavior of financial systems. There had been a variety of approaches to model such systems, especially the stock markets, such as the Langevin and the Fokker–Planck equations (Angel et al., 2006; Düring & Toscani, 2007; Wosnitza & Leker, 2014; Mariani & Liu, 2007; Chiang, Yu, & Wu, 2009), probability theory and entropy (Alfi, Coccetti, Marottad, Pietronero, & Takayasu, 2006; Donangelo, Jensen, Simonsen, & Sneppen, 2006; Wohlmuth & Andersen, 2006; Farmer, 2000; Ha, 2012; Schinckus, 2013; Wosnitza & Leker, 2014; Garzarelli, Cristelli, Pompa, Zaccaria, & Pietronero, 2014; Yin & Shang, 2014; Takahashi, Tokuda, Nishimura, & Kimura, 2014; Vogela & Saravia, 2014; Hua, Chen, Falcon, McCauley, & Gunaratne, 2015; Oh, Kima, Ahna, & Kwak, 2015), chaotic behavior, scaling relations, and fractal dimensions (Kim, Kwon, & Yook, 2013; Petković, Lončarević, Jakšić, & Vrhovac, 2014; Sarvan, Stratimirovic, Blesic, & Miljkovic, 2014; Zhong et al., 2014; Yalamova, 2012), quantum mechanics, spin models, and other models (Jiang, Chen, & Zheng, 2013; Horváth & Pincak, 2012; Krause & Bornholdt, 2013; Barad, 2014; Yuan & Ping, 2014; Nastasiuk, 2014; Sornette, 2014; Queiros, Curado, & Nobre, 2007; Dahui, Li, & Zengru, 2006; Yang, Chae, Jung, & Moon, 2006; Schulz, 2003; Takayasu, Mizuno, & Takayasu, 2006; Alfi, De Martino, Pietronero, & Tedeschi, 2007; Canessa, 2009; Tuncay, 2006; Manchanda, Kumar, & Siddiqi, 2007). An interesting thermodynamic approach was to consider the asset price as the energy and its average as temperature. The up-trends and the down-trends can be interpreted as the heating and the cooling of the market, respectively. It is also possible to define entropy within this framework (Sergeev, 2008; Zarikas, Christopoulos, & Rendoumis, 2009).

A fundamental question in financial markets and essentially in all time-series systems is whether there is a driving force behind returns. Could this force be expressed within the context of physical concepts?

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There may be a vast number of parameters of a system, but how can we amalgamate them altogether into a unified form to express by a single variable? These questions can be answered from the standpoint of the growth mechanism of physical and nonphysical entities. The world population which is a physical entity and a saving account which is a nonphysical entity both grow exponentially in time if the prevailing conditions do not change. The today's population P_1 (or money M_1) increases to P_2 (or money to M_2) in a certain time interval, and in the Aristotelian sense "today's actuality is tomorrow's potentiality". Naturally, tomorrow's value is not bound with this potential, but is also affected by a high degree of probabilistic influences. In physical processes such as heat flow (i.e. Fourier's law), mass flow (i.e. Fick's law), and electric current (Ohm's law) this potentiality principle leads to a mathematical expression between flux J and driving force X,

$$J = LX \quad \text{or} \quad X = (1/L)J \tag{1}$$

where L is a phenomenological coefficient. In the case of Ohm's law one can set, J = current, X = voltage, and L = (1 / resistance) = conductance. In this general formalism force and flux are different things, but in case of population growth or money in a saving account 'the cause replicates itself'. In chemical vocabulary, such systems are autocatalytic, that is, they convert other things into their own structure, such as female rabbits feeding on grass convert grass molecules into baby rabbits, and female foxes feeding on rabbits convert rabbit molecules into baby foxes. This phenomenon known as the Lotka–Volterra problem in ecology is a fundamental process for all multi-parameter growing systems where some of the parameters are in competition. This is actually the physical basis of nonlinear or chaotic growth where some parameters grow at the expense of some others; the simplest model equation known as the logistic equation contains two terms, one is the growth term (say 'x') and the other the control term (say '$1 - x$'). In very complex systems such as financial systems the same logic applies, because they are also autocatalytic in nature, and the rising value of a share attracts the attention of people and its value increases further. After a while if the things do not go very well, for instance if there are diminishing expectations, a decay in its asset value takes place, and the price of other assets or financial sources increase as the people will change their preferences, i.e. the autocatalytic power of other systems start to dominate the market.

For autocatalytic systems Eq. (1) can be simply written as $X_i = GX_{i+1}$ where G is a proportionality constant between the values at i'th and $i+1$'th states, and in terms of asset prices (S) it is simply,

$$S_n = GS_{n+1} \tag{2}$$

where the G term includes all probabilistic effects of the future expectations and converts today's actuality (which is the tomorrow's potentiality) to tomorrow's actuality. It stands as a kind of resistance term and the larger it is, the smaller the conversion of potentiality (i.e. S_n) into actuality (i.e. S_{n+1}).

All living and nonliving systems exhibit viscoelastic behavior, that is, the forces which affect the systems can be classified as conservative and nonconservative. The system shows reversible behavior under conservative forces, and irreversible or dissipative (i.e. entropy producing) behavior under nonconservative forces. The viscoelastic behavior of financial markets was discussed in the past, and it was shown that the components of G can be used as the modulus of elastic and viscous components of forces (Gündüz, 2009; Gündüz & Gündüz, 2010). In fact, G is a resistance (or modulus) to the acting force. Within this context, such systems can be elucidated in terms of the concepts of thermodynamics, such as work which is associated with the non-dissipative component, and heat which is associated with the dissipative component (Gündüz, 2012).

In finance, the percent asset return is simply expressed as $(S_{n+1} - S_n)/S_n = (1/G) - 1$ which is an inverse function of G. In return or logarithmic return terms G simply stands as nothing but an inverse proportionality constant. Since the traditional financial mathematics is mainly based on probability theory because of the inherent stochastic property of financial systems, G does not have any physical meaning except being a ratio.

In this research work, the meaning of Wiener noise will be elucidated in terms of G, and the dynamical behavior of financial systems will be investigated in terms of each Wiener shock that hits the asset value process with respect to G and its components. The financial dynamics will be elucidated in terms of physical concepts such as force, resistance (or modulus), energy, work-like and heat-like terms.

2. Viscoelasticity of asset prices

Asset price change is generally considered to be a stochastic process obeying the so-called geometric Brownian motion, in which the logarithm of the randomly varying quantity follows a Wiener process or as often called standard Brownian motion that has a drift term. This Wiener noise corresponds to the randomly varying stochastic component of the geometric Brownian motion. It is said that a stochastic process of asset $S(t)$ follows the geometric Brownian motion when the following stochastic differential equation is satisfied:

$$dS = \mu S dt + \sigma S \varepsilon \sqrt{\Delta t} \tag{3}$$

where μ, which represents the drift as a percentage of $S(t)$, and σ which represents the volatility as a percentage of $S(t)$, are constants. The Wiener process enters this equation as a differential random noise W_t, and simply expressed by $\Delta W = \varepsilon\sqrt{\Delta t}$. Since $\sqrt{\Delta t}$ is a constant in time-series systems, the ε is usually notated to be the Wiener noise W.

Using a drift in the model aims at modelling deterministic trends, whereas the product of the percentage volatility and the Wiener noise aims at modelling unpredictable events. In the case of no drift, Eq. (3) simply becomes,

$$\frac{dS}{S} = \sigma\varepsilon\sqrt{\Delta t}. \tag{4}$$

When written in difference form it becomes,

$$\frac{S_{n+1} - S_n}{S_n} = \sigma W_n \quad \text{or} \quad \frac{S_{n+1}}{S_n} = \sigma W_n + 1. \tag{5}$$

Using Eq. (2) one gets,

$$\frac{1}{G_n} = \sigma W_n + 1. \tag{6}$$

So, there is a hyperbolic dependence between G and W. The dependence of G to the inverse of W presents a kind of duality relation when $\sigma \gg 1$. That is, the way G behaves is depicted also by the way $1/W$ behaves, or vice versa. The duality relation well-known in string theories was discussed in financial processes on an entirely different context by Horvath (Horváth & Pincak, 2012). Note that the drift term is missing in Eq. (4) and thus in Eq. (6).

The term viscoelasticity refers to the deformation both in elastic and viscous manner. Elastic deformation can be represented by a spring or rubber band which can expand and contract with 100% recovery. A viscous deformation can be represented by a cylinder–piston system where the expanded cylinder never goes back. Most of the plastic materials exhibit both properties being partially elastic and partially viscous, and they are said to exhibit viscoelastic property. In the scientific sense the conservative systems display elastic behavior whereas the dissipative systems display the viscous behavior. If the deformation in a system is 100% recovered it is elastic, and when partially recovered it is viscoelastic. As a very simple case think of an asset price of which value increases up smoothly first but then comes back smoothly to its very original value in a day. It is purely elastic behavior. Another case is