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On Economic Space notion☆

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ABSTRACT

This paper introduces Economic Space notion to expand capacity for economic and financial modeling. Introduction of Economic Space allows defining economic variables as functions of time and coordinates and opens the way for treating economic and financial relations similar to mathematical physics equations. Economic Space allows study of economic models on discreet and continuous spaces with different dimensions. The number of risks measured simultaneously determines Economic Space dimension. We present examples of modeling on Economic Space: option pricing and derivation of Black–Scholes–Merton equation on n-dimensional Economic Space; Markov processes and derivation of Fokker–Plank Equations. Usage of Economic Space allows construing approximations of Economics and Finance similar to physical kinetics and hydrodynamics and derives Wave Equations for Economic and Financial variables.

1. Introduction

Economics and finance are systems with extreme complexity similar to complexity of theoretical physics; nevertheless the phenomena of these disciplines are too different. During the last decades Econophysics has delivered many contributions for understanding and modeling economic and financial features (Mantegna & Stanley, 2000; Roehner, 2002; Stanley, 2003; McCauley, 2006; Special Issue, 2008; Schinckus, 2013) and presented many applications of statistical physics methods to economics and finance. Methods and models developed by theoretical and statistical physics might be useful for description of economics and finance. These results present first steps for modeling economic processes with accuracy similar to and with understanding of comparable to current description of physical processes. We treat Econophysics as a way to adopt current methods and schemes of theoretical and statistical physics to economic and financial modeling. Such adoption should follow economic and financial laws and these requirements change foundation basement of most methods and models developed within theoretical and statistical physics.

We study economic analogies of physical schemes that can be useful for economic and financial modeling. One of the most general and common physical notions is the space–time issue. We state a simple question: is it possible to introduce certain analogy of space–time notion for economic and financial modeling and what are the possible advantages? Further we mention such analogy as Economic Space.

We introduce Economic Space notion as an extension of matter that is already used in economics for many years and thus we assume that Economic Space notion conforms to economic phenomenology. At the same time Economic Space notion allows regarding economic variables as functions of time and Economic Space coordinates. It allows treating economic and financial properties in a way similar to mathematical physics equations and applies methods of current theoretical physics to economic modeling. It is already well known (McCauley, 2006) and we also underline the statement that the lack of conservation laws and symmetries in economics and finance makes differences between economic and physical systems vital. Theoretical physics methods should be changed to obey economic and financial phenomenology. Space–time determines the foundation of theoretical physics and description of Nature. Thus motivation of our study is based on assumption that introduction of Economic Space may give a new look on Economics and build solid foundation for broad usage of theoretical physics methods for economic and financial modeling.

The plan of the paper is as follows. In Section 2 we argue Economic Space notion. Then we present examples of financial and economic modeling on Economic Space. In Section 3 we study option pricing on n-dimensional Economic Space and derive extension for Black–Scholes–Merton equation. In Section 4 we study Markov processes and derive Fokker–Plank equations on Economic Space. In Section 5 we study approximations of macroeconomics that are similar to physical kinetics and hydrodynamics and derive wave equations for economic variables on Economic Space. The conclusions are in Section 6.

☆ This work was performed on my own account only.

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2. Economic Space notion

We introduce Economic Space notion with the goal to study economic variables of economic agents and economic variables of entire macroeconomics as functions of time and Economic Space coordinates. Economic variables can describe Supply and Demand, Loans and Debts, Labor and Taxis, Production Function and Capital and so on. Our definition of Economic Space notion has nothing in common with spatial econometrics (Anselin, 2009) and our approach completely differs from agent-based economics (Judd & Tesfatsion, 2005). We assume that different economic states allow define different "intrinsic" Economic Spaces that describe different approximations of economic and financial processes. Existence of such Economic Space and opportunity describes economic variables as functions of time and Economic Space coordinates allows use functional analysis, stochastic functions and modern methods of mathematical physics for economic and financial modeling.

Introduction of Economic Space for economic modeling arises two problems. First: how to define Economic Space? Second: how to measure coordinates of economic variables and how to define the partition of economic variables over certain Economic Space? To solve these problems we suggest use methods that are well known in economics for many years. We refer to Lee (1999); BIS (2011) and BIS (2013) as a small amount of economic and financial risk management studies. Risk management deals mostly with risk ratings of banks and corporations. Risk ratings are provided by international rating agencies like Fitch (2006); Moody's's (2007); S&P (2012) and DBRS (2013). We state that Economic Space notion and economic and financial modeling on Economic Space can be developed on the basis of risk rating practice.

To show that let outline that risk ratings procedures distribute economic agents as companies, corporations, and banks over a finite number of risk grades, like *AAA*, *BB*, *CCC* and so on. Finite number of risk grades can be treated as finite number of points of discreet space. Risk grades of economic agents like banks or corporations can be treated as their coordinates on discreet space. Risk ratings estimations of economic agents are similar to measurements of coordinates of economic agents on discreet space. Ratings of single risk can be treated as coordinates on one-dimensional discreet space. Simultaneous estimations of ratings of n risks of economic agents are similar to measuring coordinates on n-dimensional discreet space. Thus existing risk ratings practices can be treated as procedures that distribute economic agents on discreet space. We suggest mention such space as Economic Space. Let associate risk ratings or coordinates of particular economic agent with coordinates of its economic variables. That defines economic variables of economic agent as functions on Economic Space.

Let propose that current risk ratings methodologies can be extended to estimate risk ratings of any economic agent as banks and corporations, householders and personal investors, etc. Let assume that risk methodologies may allow measure risk ratings of economic agents on discreet or continuous spaces. Risk ratings may define probability that economic agent has certain coordinates on discreet or continuous space. Let study risk ratings of economic agents that are under simultaneous action of n risks on Euclidian space R^n. Location of economic agent on Economic Space R^n, or the risk grade of economic agent can be defined by probability distribution.

Let define Economic Space as any mathematical space that is used to map economic agents by their risk ratings as space coordinates. Further let study economic modeling on Economic Space R^n.

To describe economics on Economic Space R^n one should determine n risks that disturb economics now. It seems impossible to take into account all existing risks that can affect current economics. Risk ratings procedures contain internal uncertainty and that uncertainty will grow up with growth of number of simultaneously measured risks. If one takes into account too many different risks then simultaneous measurements of all these risk ratings will have too high variability and hence model description can be too uncertain. To determine reasonable Economic Space one should estimate current risks and select two, three, four most important risks as main factors affecting contemporary economics. That allows define Economic Space that has dimension two or three and derive appropriate initial distributions of economic agents and their economic variables. To select most valuable risks one should establish procedures that allow comparison of influence of different risks on economic processes and determines initial state of Economic Space R^n.

To describe economic evolution in time term T it is necessary to foresee m main risks that will play a major influence on economics in particular time term and define Economic Space R^m. The set of m risks can be the same as for initial state, or a different one. This set of m risks defines target state of Economic Space R^m.

Then it is necessary define transition dynamics, transition model that describes move from initial set of n main risk and define initial representation of Economic Space R^n to target set of m main risk and target representation of Economic Space R^m. Such transition model describes how new risks grow up and how initial set of risks decline its action on economic processes. Transition dynamics from initial set of n main risk to target set of m risks describes evolution of initial representation R^n to target one R^m.

And to complete the list of problems we outline that one should develop model of evolution of economic distributions under action of initial set of n risks, develop a model of evolution under target set of m risks and describe transition dynamics from initial risks to target risks.

That short description arises a lot of difficult problems. The selection of main risks simplifies economic description and allows neglect "small risks". On the other hand the selection process becomes a part of validation procedure. As one can select and measure main risks, then it is possible to validate initial and target set of risks and prove or disprove initial model assumptions. The procedures that measure and compare influence of different risks on economics should be determined and that is a separate tough problem.

Transition process from initial set of risks to target risks becomes measurable. Hence it might be possible compare transition observations with model assumptions and estimate accuracy of model predictions. Financial modeling and forecasting on Economic Space are splitting into set of verification procedures. It gives a chance to make financial modeling more measurable and it's forecasting more faithful. To prove that one should establish reasonable modeling. The success or failure will be best argument. With this in mind we move forward in developing modeling on Economic Space.

To demonstrate possible advantages of Economic Space notion usage for economic and financial modeling we present some cases. Further we assume that methodologies and procedures that allow measure risk ratings of economic agents, as their coordinates on n-dimensional Economic Space R^n exist.

3. Option pricing on Economic Space

As first example of financial modeling on Economic Space we present option pricing. Option pricing theory is based on the Black–Scholes–Merton equation (BSM) (Black & Scholes, 1973; Merton, 1973; Merton, 1998) and that is one of the most recognized equations in financial theory. We present an extension of the BSM equation on n-dimensional Economic Space. Further we shall mention Economic Space as e-space.

The BSM equation for price V of option on underlying asset with price a has form:

$$\frac{\partial V}{\partial t} + ra\frac{\partial V}{\partial a} + \frac{1}{2}\sigma^2 a^2 \frac{\partial^2}{\partial a^2} V = rV \tag{1}$$