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Vine copulas and applications to the European Union sovereign debt analysis

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ABSTRACT

European sovereign debt crisis has become a very popular topic since late 2009. In this paper, sovereign debt crisis is investigated by calculating the probabilities of the potential future crisis of 11 countries in the European Union. We use sovereign spreads of the European countries against Germany as targets and apply the GARCH based vine copula simulation technique. The methodology solves the difficulties of calculating the probabilities of rarely happening events and takes sovereign debt movement dependence, especially tail dependence, into consideration. Results indicate that Italy and Spain are the most likely next victims of the sovereign debt crisis, followed by Ireland, France and Belgium. The UK, Sweden and Denmark, which are outside the euro area, are the most financially stable countries in the sample.

1. Introduction

The ongoing European sovereign debt crisis originated in Greece, but the impact has spread all over the European Union especially in the euro area. On 8th Dec, 2009, rating agency Fitch cut Greece's long-term debt from A− to BBB+. Because of the lack of confidence in investing in Greek government bonds, the yield of 10-year government bonds jumped up significantly. In the mean time, the bond yield of peripheral European countries Spain and Portugal also increased along with Greece. In Ireland and Italy, however, the yields decreased. This phenomenon shows that yield differentials across European bond markets have not been wiped out completely, although accelerated financial integration among euro bond markets has been widely expected, since the macroeconomic and fiscal indicators have shown significant improvement for the higher risk euro markets, creating a potential for those members to converge with lower risk members in terms of bond returns. Finding the relationship between the yields of these countries' sovereign bonds might be a useful way to understand how they will influence each other, especially in extreme events. This information could then be used to assess the risk level of a sovereign bond. In order to achieve this, a GARCH based vine copula simulation method to analyze the sovereign debts in the European Union is proposed in this paper.

As a popular multivariate modeling tool, copula is widely used in many fields where the multivariate dependence matters, such as actuarial science (Frees, Carriere, & Valdez, 1996), biomedical studies (Wang & Wells, 2000), engineering (Genest & Favre, 2007) and finance (Embrechts, Lindskog, & McNeil, 2003). In finance, the misuse of the copula method in the pricing of collateralized debt obligations (CDO) is considered by journalists to be one of the reasons that led to the global financial crisis of 2008–2009 (Jones, 2009, April 24; Salmon, 2009, February 23). The copula approach provides a method of isolating the description of the dependence structure and understanding the dependence at a deeper level. It expresses dependence on a quantile scale, which is useful for describing the dependence of extreme outcomes and is natural in a risk-management context. Due to the advantages of the copula method, it is an ideal tool for analyzing the relationship of sovereign debts between countries in the European Union.

The main difficulty about sovereign debt crisis analysis is that the crisis rarely happens. It is extremely hard for statisticians to analyze an event which has never happened before. In order to solve this issue, this paper uses simulation methods to create unknown situations. This paper replicates 10000 iterations of a 365 future day simulation of sovereign spreads against Germany of 11 countries in the European Union. In the mean time, the relationships between the countries are considered. Then, the percentage chance of the crisis events is calculated, which is the probability of future crisis. In terms of defining crisis events, Sy (2004)'s definition of sovereign debt crisis is adopted, which is that sovereign spread against the US is more than 1000 basis points. In the same manner, a country experiencing a sovereign debt crisis is defined as being when its sovereign spread against Germany is greater than 1000 basis points in this research.

The contribution of this research is fourfold: firstly, this is the first analysis of extreme value and tail dependence of sovereign debt spread movement in the European Union; secondly, this study conducts the comparison between 11 countries in the European Union at the same time; thirdly, this paper uses vine copulas to deal with large numbers

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of dimensions and satisfies the wide range of dependence, flexible range of upper and lower tail dependence, computationally feasible density for estimation, and closure property under marginalization simultaneously; fourthly, which is also the key feature of this paper, the research identifies the risk level of sovereign debt in different countries in the European Union.

Daily 10-year government bond yields from 18/06/1997 to 12/03/2012 in Belgium, Denmark, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden and the UK are used in this research.

The results show that the estimated crisis probabilities of Greece and Portugal in the next 365 days are 100% and 99.77%, which is consistent with the situation that they are already in crisis. Spain and Italy show great potential to be the next victims in one year's time. France and Belgium show some instability in the results and the probability of crisis is fairly high: 63.13% and 60.14% respectively. Netherlands is next with an almost 1 in 4 chance of crisis and it is the most stable country in the euro area. In the mean time, countries outside the euro area in the sample which are the UK, Sweden and Denmark show the greatest stability in their sovereign bonds.

The remainder of the paper is as follows. Section 2 is a literature review in sovereign debt analysis and copula methods. Section 3 is the data description. Section 4 discusses the bivariate relationships of these pairs of countries. Section 5 explains the vine copula approach. Section 6 shows the results of simulation and calculation of the risk levels of the countries. And Section 7 concludes.

2. Literature review

The literature on sovereign debt analysis generally uses sovereign bond spread between a target country and a benchmark country to assess the default risk level of the target country. Structural approaches developed from the Merton model (Merton, 1974) and reduced form models such as the Jarrow and Turnbull (1995) approach are the two main streams.

The structural approaches explain the sovereign spread endogenously using both enterprise value volatility and firm default definition. The pitfalls of these approaches are not only their difficulty and lack of accuracy to define appropriate country-specific proxy variables for the level of indebtedness, but also they disregard the fact that default incentives of a country are more complicated than those of enterprises. The reduced form approaches use different macro variables as the determinants of the sovereign default risk. Literature such as Reinhart, Rogoff, and Savastano (2003), Eichengreen, Hausmann, and Panizza (2003) and Goldstein and Turner (2004), analyze the sovereign debt risk of emerging market economies. Their focus is on the sustainability of the sovereign debt and the currency mismatches. They measure default risk by using country credit ratings. The disadvantage of these approaches is that these credit ratings are inefficient and cannot be adjusted in a timely manner to adapt to the market data when a big crisis is ongoing. Most recently, Dötz and Fischer (2010) use a GARCH-in-mean based reduced form model to analyze the factors triggering the sovereign spread movement in the European Union and the result shows that the expectation of loss is the main reason sovereign spreads widened during the recent global financial crisis. Nonetheless, both structural and reduced form approaches face a problem: they ignore the yields movement dependence with other countries, which is especially important inside the European Union.

Both multivariate extreme value theory (EVT) and copula method can solve these problems in order to capture the probabilities of rare events. Multivariate EVT is developed by de Haan and Resnick (1977) for limiting distribution of the component-wise maximum of independent and identically distributed (i.i.d.) random vectors. The technique has since been comprehensively developed. Although in the literature multivariate EVT is available in a d-dimensional context, the computational complexity increases significantly with the increase of d (Fougères, 2003). For instance, applications which are done by de Haan and de Ronde (1998), Bruun and Tawn (1998), and de Haan and Ferreira (2006) as with most work done in the multivariate EVT context, are limited to 2 and 3 dimensions. With reference to copula method, there is a large body of literature using copulas in a financial context (Bouyé, Durrleman, Nikeghbali, Riboulet, & Roncalli, 2000; Cherubini, Luciano, & Vecchiato, 2004; Embrechts et al., 2003). Most of them are used to compute Value at Risk (VaR) and expected shortfall (ES) of the stock or bond portfolio by applying single copula families such as elliptical copulas and Archimedean copulas. There are many limitations on those copula families applied in the above literature. Elliptical copulas are widely used, but they cannot model the financial tail dependence very well (Patton, 2008). Archimedean copulas are not satisfactory for modeling with dimensions higher than two (Joe, 1997). Multivariate Archimedean copulas only allow exchangeable structure with a narrower range of negative dependence in a higher dimension (McNeil & Neslehova, 2009). Partially symmetric copulas extend Archimedean to a class with a non-exchangeable structure, but the dependence they provide are not particularly flexible (Joe, 1993). Mix-id copulas in Joe and Hu (1996) provide flexible positive dependence by construction, but only upper tail dependence is flexible not lower tail. Demarta and McNeil (2005) provide multivariate skewed-t copulas, which model well, but are computationally more involved. Similarly to multivariate EVT, these copula methods experience limitation about dimension. Vine copulas were proposed by Joe (1996) and explained in detail by Bedford and Cooke (2002). At that time, vine copulas models were a graphical model using bivariate copulas to construct multivariate copulas. Aas, Czado, Frigessi, and Bakken (2009) run statistical inference on two types of vines: canonical vine (C-vine) and drawable vine (D-vine). These models have been improved by Nikoloulopoulos, Joe, and Li (2012) which can satisfy most of the features that should be included in a copula model: firstly, a wide range of dependence including both positive and negative dependence; secondly, a flexible range of upper and lower tail dependence; thirdly, and most importantly, computationally feasible density for estimation, even for high dimension estimation. According to the aim of this analysis, which is focusing on the interactions of the 11 countries and assessing the crisis probabilities of countries simultaneously, vine-copula method is preferred to other models above.

In this paper, a GARCH based Vine copula method is used to analyze the tail dependence and calculate probabilities of sovereign debt crisis of these countries in certain periods of time in the European Union.

3. Data

Daily 10-year government bond yields from 18/06/1997 to 12/03/2012 in Belgium, Denmark, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden and the UK are used in this research. All data are collected from Thomson Reuters ECOWIN. The target variable, sovereign spread against Germany, is calculated as

$$\Delta\left(i_j - i^*\right),$$

where $j = 1,\ldots,d$, i_j is 10-year government bond yield of a target country, and i^* is 10-year government bond yield of Germany.

4. Bivariate copula analysis

4.1. GARCH filter

Vine copula modeling proceeds in three stages. In the first stage, the model for the individual variables (i.i.d) is selected, which is the marginal distribution. For financial time series data, a GARCH

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