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Focusing on the worst state for robust investing

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ABSTRACT

Despite its shortcomings, the Markowitz model remains the norm for asset allocation and portfolio construction. A major issue involves sensitivity of the model's solution to its input parameters. The prevailing approach employed by practitioners to overcome this problem is to use worst-case optimization. Generally, these methods have been adopted without incorporating equity market behavior and we believe that an analysis is necessary. Therefore, in this paper, we present the importance of market information during the worst state for achieving robust performance. We focus on the equity market and find that the optimal portfolio in a market with multiple states is the portfolio with robust returns and observe that focusing on the worst market state provides robust returns. Furthermore, we propose alternative robust approaches that emphasize returns during market downside periods without solving worst-case optimization problems. Through our analyses, we demonstrate the value of focusing on the worst market state and as a result find support for the value of worst-case optimization for achieving portfolio robustness.

1. Introduction

Because diversification is critical as it is known to reduce idiosyncratic risk in the theory of portfolio selection, there has been a substantial amount of research published that seeks to reconcile diversification benefits with investment practice. Diversification lowers portfolio risk by investing in assets that move independent to one another; that is, it takes advantage of assets with low correlation. Unfortunately, it has been reported that correlation across different financial instruments increases during market crashes. The equity market is no exception as correlations among international equity markets increase in bear markets (Campbell, Koedijk, & Kofman, 2002). In fact, Yang and Bessler (2008) show international stock market contagion around the October 1987 crash. Furthermore, Silvapulle and Granger (2001) use 30 Dow Jones Industrial stocks to confirm higher correlation in the U.S. market when market returns are negative. This has a significant implication on portfolio performance: investors cannot benefit from the diversification effect when it is most needed. To make things worse, the correlation within the equity market has been increasing over recent periods.

One of the main problems that can be caused by the high equity correlation with asymmetric market behavior is the non-robustness in the efficient portfolios generated by the model (see, for example, Chopra & Ziemba, 1993). This issue is due to the nature of optimization techniques, the mathematical tools for equity portfolio allocation. A proposed allocation by an optimization model is chosen from the extreme points within the feasible region, so it can be sensitive to input parameters. Since the input variables estimated from historical data may not correctly reflect the future market environments, the actual investment performance could be unstable. As equities during market crashes show significantly higher correlations, traditional portfolio optimization models (Markowitz, 1952) based on symmetric return distribution assumptions might not be suitable to protect investors from the extreme downside in the equity market.

In order to overcome this difficulty, researchers have proposed various robust portfolio optimization models. There are two popular approaches. The first approach is to employ input variables less sensitive to the historical data. This includes equal-weighted portfolio, minimum-variance portfolio, shrinkage estimation[1] and Bayesian approaches as proposed by Black and Litterman (1990). The second

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[1] See Jobson and Korkie (1981), Jorion (1986, 1991), Pastor (2000), and Larsen and Resnick (2001).

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approach employs modified analytical models whose optimal solutions are robust to the input variables.[2] Major efforts have been made to link traditional portfolio allocation models with advanced robust optimization techniques (Goldfarb & Iyengar, 2003; Lobo & Boyd, 2000; Tütüncü & Koenig, 2004). The implementation of this second approach involves first defining uncertainty sets on the input parameters, and then reformulating the models into tractable optimization problems in order to obtain the worst-case optimal solution (max–min solution) while ensuring a certain level of computational efficiency.[3]

Interestingly, however, most of the research effort related to the second approach has been made to develop mathematical models for optimizing the worst case by reformulating the problem with various assumptions, rather than to understand the relationship between market behavior and worst-case optimization. Robust portfolios are formed using worst-case optimization, but does optimizing the worst case really result in a portfolio that is more robust? Optimizing the worst case is intuitively understandable, but can it be simply applied to portfolio allocation without first understanding the true value of the information obtained in the worst market situations? In this paper, we demonstrate the importance of focusing on worst cases for achieving portfolio robustness, which explains why worst-case optimization is a valid approach for robust portfolio construction. Our attention in this paper will be on the stock market since most research in worst-case optimization for portfolio selection focuses on robust equity portfolios. In order to analyze the worst-case behavior, we assume that the stock market can be divided into several states, each with its own characteristic. We find the value of focusing on the worst state for robust investing through the following chain of reasoning. We first show that the optimal choice of portfolio in a multi-state environment is the portfolio with robust expected return. We then look at tail events in the stock market and find that placing more emphasis during the worst state results in portfolios with more robust returns. We further show how robust portfolios can be formed by focusing on the worst market periods even without directly solving worst-case optimization problems to justify the value of using worst-case information. These results allow us to understand the reasoning behind why worst-case optimization leads to robust portfolios and provide evidence that robust models should incorporate worst-case information into the models.

The remainder of this paper is organized as follows. Section 2 analytically describes the optimal portfolio under multiple market states. Section 3 describes key characteristics of the stock market that allow one to achieve robust investment performance and Section 4 identifies the worst state as the most valuable state for achieving robust performance. New approaches for forming robust portfolios along with the empirical results and their implications are shown in Section 5. Section 6 summarizes and concludes the paper.

2. Optimal portfolio under multiple states

In order to analyze the worst-case behavior, we assume that the market can be divided into several states, each with its own characteristics. This assumption of multiple states in the stock market is not a novel approach (see, for example, Chu, Santoni, & Liu, 1996; Schaller & Van Norden, 1997; Turner, Startz, & Nelson, 1989), and there also have been efforts to study portfolio selection with the information of multiple regimes. Ang and Bekaert (2002) study a dynamic portfolio choice problem that accommodates time-varying correlations and volatilities due to the regime-switching behavior in the international equity market. Honda (2003) and Zhou and Yin (2003) develop a continuous-time version of the mean-variance model with regime switching modulated as a Markov chain. Similarly Çelikyurt and Özekici (2007) look at several multi-period portfolio optimization models in a stochastic market also represented as a Markov chain.

In this context, we also assume the stock market to have several distinct states. Nevertheless, since defining and detecting market states is a broader domain, we focus on the situation when we only know the existence of states and the number of states in the market. Furthermore, we define a robust portfolio as a static portfolio where the portfolio outcome is not affected by the underlying states. Since market behavior is dissimilar during different market states, our goal is to find an optimal static portfolio given that there are N states in the market without knowing the details of each state. We find that the optimal choice of portfolio in a market with multiple states is the portfolio whose expected return is not affected by the future market state.

Proposition 1. *When multiple states exist in the market, the optimal static mean-variance portfolio for a risk-averse investor is the robust portfolio where the expected portfolio return is constant in all states.*

Proof. See Appendix A.

This implies that the portfolio that performs better when the existence of states is known is the robust portfolio that has robust returns. This gives additional motivation for constructing robust portfolios because robust portfolios will not only provide robust performance but also optimal performance. Hence, we further illustrate forming robust portfolios using worst-case information in the following sections.

3. Which market states matter in robust asset allocation?

We now analyze different stock market states and identify states that show distinct behavior from the average market movements because these states will be the most important in forming robust portfolios. As noted earlier in this paper, the typical approach currently employed in robust portfolio construction is to modify a traditional portfolio model such as the Markowitz model into a different version which is less sensitive to the input data. While such an approach is meaningful, it does not place much emphasis on stock market behavior. In other words, if there is a market state that is critical to investment performance and under which the market behavior becomes dramatically different from other states, robustness in the portfolio performance could be even more improved by treating the state with special care, on top of employing the robust models.

In order to identify such a state, we recall the asymmetry of the stock return distribution. It is widely known that stock returns are negatively skewed, meaning that there are more extreme returns in the left tail than the right tail. Table 1 clearly captures the effects of the negatively skewed return distribution to the investment performance. Using daily returns from 1973 to 2011, the annualized return on the U.S. stock market index was 9.82% and the final wealth with an initial investment of $1 was $38.61. If the investor could avoid the return on the worst day at the cost of realizing the return on the best day among 10,174 daily returns, the annual return increases to 10.10%, providing an additional 28 basis points compared to the investment performance without truncating the extreme events on both sides (i.e., the unconditional investment performance). Similarly, if the five worst returns (five worst days) at the cost of the same number of the best returns (five best

Table 1
Investment performance of the U.S. market from January 1, 1973 to December 31, 2011.

	Annual return	Difference	Total WEALTH
Datastream Market Index	9.82%		38.61
Exclude returns on worst and best days	10.10%	0.28%	42.57
Exclude returns on 5 worst and best days	10.23%	0.41%	44.59
Exclude returns on 39 worst and best days	10.90%	1.08%	56.50

[2] Mulvey, Vanderbei, and Zenios (1995) introduce a general formulation of robust optimization and describe how it differs from sensitivity analysis and stochastic linear programming.

[3] For further details, see Fabozzi, Kolm, Pachamanova, and Focardi (2007a,b), Fabozzi, Huang, and Zhou (2010), and Kim, Kim, and Fabozzi (2014).