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Calculating and comparing security returns is harder than you think: A comparison between logarithmic and simple returns

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ABSTRACT

We analyse the relationships between return calculation methods, risk and observation periods. We show that the mean of a return set calculated using logarithmic returns is less than the mean calculated using simple returns by an amount related to the variance of the set. This implies that there is not a one-to-one relationship between mean logarithmic and mean simple returns and also that risk and return calculations are not independent as the measure of risk is part of the measure of return. Finally we draw on examples from the extant literature to illustrate that these effects can be very important particularly when dealing with short observation periods.

1. Introduction

In this paper we analyse the relationships between return calculation methods, risk and observation periods. Two return calculation methods are very commonly used in finance, with both logarithmic and simple returns being calculated routinely,[2] although there has been surprisingly little discussion of the differences between the methods. We show that there are some theoretically interesting and sometimes substantial differences between mean returns calculated using logarithmic returns and those calculated using simple returns.

We show that the mean of a set of returns calculated using logarithmic returns is less than the mean calculated using simple returns by an amount related the variance of the set of returns where the variance is relatively invariant whether it is measured using logarithmic or simple returns. One implication of this is that there is not a one-to-one relationship between mean logarithmic and mean simple returns.[3] Given this, it is clearly unsound to compare the conclusions of studies done using different return measures without considering this factor. Thus, for example, it is not possible to extrapolate conclusions about terminal wealth from studies carried out using logarithmic returns. In particular, if period 1 has a higher mean logarithmic return than period 2 this does not necessarily imply that the mean simple return in period 1 is higher than in period 2. Thus even the most basic qualitative conclusions derived from investigations using logarithmic returns may not hold for the monetary returns of actual investments. Another implication is that, given the mean calculated return in a period depends on the variance of returns in that period, the risk and return in that period are not independent by construction which is troubling in the context of much finance theory.

This relationship between variance and return does, however, enable the derivation of an approximate method for converting between means calculated using logarithmic returns and those calculated using simple returns. This result can enable meaningful comparisons to be made between past empirical studies made using alternative returns measures.

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[2] In this paper we adopt the following notation:
$R_{Lt} = \ln(P_{t+1}) - \ln(P_t)$ where R_{Lt} is the log return for period t, P_{t+1} is the price of a security at time $t+1$ and P_t is the price of a security at time t.
$R_{St} = P_{t+1}/P_t - 1$ where R_{St} is the simple return for period t, P_{t+1} is the price of a security at time $t+1$ and P_t is the price of a security at time t.

[3] In mathematical terms there is a non-injective relationship between mean logarithmic and mean simple returns.

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We undertake empirical studies to confirm the theoretical findings discussed above. We illustrate that the direct relationship between risk and return depends on how returns are measured by using a GARCH-M model. We then assess the relationship between risk and return for logarithmic and simple returns in periods of differing variance.

The paper further illustrates, by means of practical examples using cases drawn from the extant finance literature, that there can be a substantial empirical difference between results derived using logarithmic returns and those derived using simple returns and confirms the circumstances in which these differences are likely to be most important. Much of the literature in finance is, of course, related to calculations and comparisons of security returns so it is not possible to give a comprehensive range of examples. We do, however, cover a range of areas. First, we draw an example from the research into calendar based anomalies where a time series is divided into subsections based on a particular calendar effect such as the day of the week, month of the year, the day before a public holiday etc. Second, an example is taken from the research into trading rules in which a time series of security returns is divided into subsections that are expected to exhibit different returns using a particular trading rule. Third, an example is drawn from the literature on event studies where a time series is divided into subsections based on when specified events take place. Examples of such events include stock splits, IPO's, results declarations and other corporate events and other market events such as large drops in stock prices. Over these areas, there seems little consensus or indeed discussion in the literature regarding the best method of calculating returns and indeed many papers do not clearly specify which type of return is used. Even a moderate level of investigation, however, gives the conclusion that each of the literatures mentioned freely uses both logarithmic and simple returns and we provide evidence to support this assertion.

In the penultimate section of the paper we consider how conclusions from finance research can depend crucially on the return measure used. We look at the conclusion from published research studies using daily data. Finally we carry out an investigation showing how different return measures can have a very substantial effect on results when intraday data is used.

This paper has the following structure: Section 2 discusses the features of logarithmic returns by reference to simple returns; Section 3 analyses the relationship between logarithmic and simple returns and also derives an approximate method for converting between means calculated using logarithmic returns and those calculated using simple returns; Section 4 considers the implications of the way that returns are calculated in the direct measurement of the relationship between risk and return; Section 5 discusses the implications of the way that returns are calculated in the context of the literature comparing security returns in different time periods, Section 6 investigates the practical importance of using different return measures and Section 7 presents the conclusions.

2. Discussion of the features of logarithmic returns by reference to simple returns

Calculating the return on a security in a particular period as the difference between the natural logarithm of the security price at the end of the period and the natural logarithm of the security price at the beginning of the period (referred to as a logarithmic return) is a very commonly used procedure in finance even though this returns differs from the monetary growth which would be actually be achieved by an investment over that period (which is measured by the simple interest over that period). A number of strong arguments are put forward to justify the use of logarithmic returns:

i) Logarithmic returns can be interpreted as continuously compounded returns. This means that, for non-stochastic processes, such as the returns on risk-free fixed interest securities held to maturity, when logarithmic returns are used, the frequency of compounding does not matter and returns across assets can more easily be compared.
ii) Using continuously compounded (logarithmic) returns is advantageous when considering multi-period returns as the continuously compounded multi-period return is simply the sum of continuously compounded single period returns. Continuously compounded returns are time additive and it is easier to derive the time series properties of additive processes than multiplicative processes (see Campbell, Lo, & Mackinlay, 1997, p 11). In this context some studies have shown that using simple returns to estimate returns over longer periods can be quite unsatisfactory (see, Dissanaike, 1994 and Roll, 1983).
iii) The use of logarithmic returns prevents security prices from becoming negative in models of security returns (see Jorion, 2001, p 100).
iv) If a security price follows geometric Brownian motion[4] (a very popular model of security price movements used, for example, in the Black–Scholes option pricing model) then the logarithmic returns of the security are normally distributed.
v) For forecasting future cumulative returns, continuous compounding of the expected logarithmic return will give a better guide to median future cumulative returns (the return that investors are likely to realise) than compounding expected simple returns (Hughson, Stutzer, & Yung, 2006).
vi) Logarithmic returns are approximately equal to simple returns. Inspection of the formula connecting logarithmic and simple returns $R_{Lt} = \ln(1 + R_{St})$ shows that as long as R_{St} is not too large (Roseff and Kinney, p 380, suggest $R_{St} \le 0.15$) then logarithmic and simple returns are very similar in size. Whilst this is true, it is important not to wrongly deduce from this that the mean of a set of returns measured using logarithmic returns is necessarily very similar to the mean of the same set of returns measured using simple returns. The theory behind this result is outlined in the next section and Appendix A.

In many areas of academic finance the advantages of using logarithmic returns appear to have been tacitly accepted although a few papers have pointed out pitfalls in their use in particular fields of investigation. In the area of event studies Dissanaike and Le Fur (2003) point out problems with the use of cross-sectional averages of logarithmic returns. Kothari and Warner (1997) and Barber and Lyon (1997) show that logarithmic returns are negatively skewed such that test statistics are unlikely to be well specified. In the area of assessing investment returns over long periods of time there has been a debate over whether logarithmic or simple means are most appropriate to assess returns (see Jacquier, Kane, & Marcus, 2003; McLean, 2012).[5]

3. The relationship between simple and logarithmic returns

This section analyses the relationship between simple and logarithmic returns. Notwithstanding the advantages described in the previous section one drawback of logarithmic returns is that they do not give a direct measure of the change in wealth of an investor over a particular period. By definition, the appropriate measure to use for this purpose is the simple return over that period. For non-stochastic systems converting between the two measures is trivial as there is a one-to-one correspondence between logarithmic returns and simple returns P.[6] The situation is much more problematic for stochastic systems as discussed in i) and ii) below.

[4] Also known as the multiplicative random walk see Cootner (1964) and Fama (1965).
[5] Jacquier et al. use different terminologies referring to geometric and arithmetic means as opposed to logarithmic and simple means.
[6] $R_{Lt} = \ln(1 + R_{St})$, $R_{St} = \exp(R_{Lt}) - 1$.