International Review of Financial Analysis 38 (2015) 163–174

Contents lists available at ScienceDirect

International Review of Financial Analysis

Trading death: The implications of annuity replication for the annuity puzzle, arbitrage, speculation and portfolios☆

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ARTICLE INFO

Article history:
Received 10 March 2014
Received in revised form 10 October 2014
Accepted 19 October 2014
Available online 27 October 2014

JEL classification:
G12
G22
G23

Keywords:
Annuities
Annuity puzzle
Arbitrage
Speculation
Life insurance

ABSTRACT

Annuities are perceived as being illiquid financial instruments, and this has limited their attractiveness to consumers and their inclusion in financial models. However, short positions in annuities can be replicated using life insurance and debt, permitting long positions in annuities to be offset, or short annuity positions to be created. The implications of this result for the annuity puzzle, arbitrage between the annuity and life insurance markets, and speculation on expected longevity are investigated. It is argued that annuity replication could help reduce the annuity puzzle, improve the price efficiency of annuity markets and promote the inclusion of annuities in household portfolios.

1. Introduction

It is widely understood that annuities cannot be liquidated or traded, and so are highly illiquid assets. This paper explores the uses of a strategy to replicate a short annuity position which allows annuitants to fully or partially offset a long annuity position, in contradiction to the accepted view. The implications of this replication strategy for the well known annuity puzzle,[1] arbitrage between the annuity and life insurance markets, speculation on a change in an annuitant's longevity and constructing household portfolios are investigated. This is one of the first studies to conduct a detailed analysis of annuity replication and to investigate annuity arbitrage and longevity speculation. It also argues that annuity replication allows annuities to be integrated into the well-known models of mainstream finance, such as portfolio theory.

Section 2 explains how annuities are priced, Section 3 covers the reasons for the illiquidity of annuities, and Section 4 presents the relationship between the actuarially fair prices of annuities and life insurance. Section 5 shows how short positions in annuities can be replicated using life insurance and debt, and Section 6 considers the implications of annuity replication for the annuity puzzle and argues that, by encouraging consumers to buy annuities, it could help solve this puzzle. Section 7 has a detailed analysis of annuity arbitrage, which is possible for annuity underpricings but not annuity overpricings. Examples are provided from a range of countries of situations where annuities have been substantially under-priced, followed by an examination of the effects of variations in annuity and life insurance load factors on arbitrage profits. This indicates that annuity arbitrage appears, on occasion, to be profitable. Section 8 demonstrates how annuity replication can be used to speculate on a change in an annuitant's expected longevity. The profits from such speculation are available in cash once the increase in expected longevity has been recognised. Section 9 argues that the ability to replicate short positions in annuities means that annuities can take a more prominent role in household portfolio decisions, and Section 10 has the conclusions.

2. Annuity prices

Annuity prices depend primarily on the expected longevity of the annuitant and interest rates expected over the remaining life of the

☆ Thanks for comments on earlier drafts to an anonymous referee of this journal, and David Blake, Lars Bovenberg, Edmund Cannon, Martin Lehmann, Geng Li, Paula Lopes, Emmanouil Platanakis, Ian Tonks and participants in the 11th International Workshop on Pensions, Insurance and Saving (Paris), the Netspar International Pension Workshop (Frankfurt) and a seminar at Reading University.

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[1] The theory (Davidoff et al., 2005; Yaari, 1965) indicates that consumers should annuitize most of their wealth, but actual rates of annuitization are much lower, and this widespread under-annuitization creates the annuity puzzle.

http://dx.doi.org/10.1016/j.irfa.2014.10.010
1057-5219/

annuitant. Annuity prices may also be affected by load factors such as adverse selection,[2] administration, regulatory and marketing costs, profit, the risk premium etc., but these effects are usually of modest size and exhibit only small variation over time. Studies for a range of countries have found that the money's worth ratio (MWR)[3] is usually not far below one and reasonably stable over time, James and Song (2001), James and Vittas (2000), Murthi, Orszag, and Orszag (1999, 2000), Finkelstein and Poterba (2002), Cannon and Tonks (2004, 2008, 2009, 2011), and Von Gaudecker and Weber (2004). Therefore load factors are not of major importance, and the prices of annuities are determined primarily by longevity and interest rate expectations.

The actuarially fair price for a single premium, instantaneous, level, single life annuity in the absence of load factors is:

$$V_x = \sum_{i=1}^{n} \frac{A \cdot {}_iP_x}{(1+r)^i} \tag{1}$$

where

- V_x is the current price or value of the annuity for an annuitant aged x years.
- i is the number of years since the annuity was purchased
- r is the rate of interest for the remaining life of the annuitant (a non-stochastic interest rate with a flat term structure is assumed)
- A is the constant annual annuity payment
- n is a number greater than the remaining years of life of the annuitant
- ${}_iP_x$ is the probability that an annuitant aged x when the annuity was purchased survives for at least i years.

As longevity expectations increase, annuity prices increase, while as interest rate expectations increase annuity prices decrease. The term ${}_iP_x$ in Eq. (1) shows that the price of an annuity depends, not on the expected longevity, but on the cumulative probability distribution of expected longevity. Therefore, any change in the longevity distribution which alters the values of ${}_iP_x$ will affect the annuity price. For example, it is possible that, although some values of ${}_iP_x$ drop, others rise, so that overall the annuity price rises.

3. Illiquidity and annuities

In the 18th and 19th centuries speculators were allowed to purchase annuities on the lives of nominees, rather than on their own lives. This permitted wide-scale adverse selection against annuity providers as annuity prices did not reflect the nominee's age or gender. For example in the early 1770s banks in Geneva created lists of Genevan young girls from families with a record of health and longevity who could serve as nominees. The preference was for girls between the ages of five and ten years who had survived smallpox (Velde & Weir, 1992). These annuities and the income they brought could be traded while the nominee remained alive. To reduce the opportunities for adverse selection, annuities can now only be purchased on the life of the annuitant, and cannot be surrendered back to the provider or traded. For example, if allowed, annuitants would seek to sell their annuity shortly before their death (Brown, 2002). Furthermore, the judgement in 1745 by Lord Chancellor Hardwicke in the case of *Lawley v Hooper* made clear that annuities which could be redeemed or repurchased were likely to be classified as usurious loans (Comyn, 1834). Although UK laws against usury were repealed in 1854, this judgement created a situation in which annuity agreements did not contain redemption clauses. Ogborn and Wallas (1955) and Browne, Milevsky, and Salisbury (2003) suggest that the current inability to surrender annuities is because insurance companies use annuity receipts to make long term illiquid investments.

Defined benefit pensions lead to an annuity supplied by the pension scheme. On retirement, members of UK defined benefit pension schemes can take up to 25% of the total value of their accrued pension liability (depending on the rules of the scheme) as a tax-free lump sum, with the remaining 75% taken as a pension. Subsequently, these annuities cannot be traded or liquidated, and so in this respect are comparable to annuities purchased from an insurance company. Similarly, many people have state pensions which cannot be traded or liquidated, and so face the same illiquidity problems.[4] Therefore, due to defined benefit pensions, state pensions and defined contribution pensions taken before April 2015, part of the wealth of most people is tied up in an annuity that cannot be liquidated or traded. Annuitants cannot borrow using their annuity payments as collateral as annuity payments are conditional on the annuitant being alive. Nor can they assign the right to receive annuity payments.

4. The relationship between annuity and life insurance prices

There is a widespread view that annuities cannot be reversed, and so are illiquid assets, which discourages their purchase. However, given various assumptions, it is possible to replicate the cash flows of a short position in an annuity, and so effectively offset or liquidate an annuity. The strategy for replicating annuities relies on the relationship between the prices of annuities and life insurance. The purchaser of an annuity makes a large initial payment, and then receives a stream of payments until death, while the purchaser of life insurance makes a stream of life insurance premia payments until their death, at which time their estate receives a large terminal payment.[5] So, apart from the large payment being made at the end rather than the start, the pattern of cash flows of a life insurance policy is the inverse of those of an annuity.

Ignoring load factors, the actuarially fair price of life insurance (i.e. the annual life insurance premium, Y) occurs when the following condition is met:

$$\sum_{i=1}^{n} \frac{{}_iD_x X^*}{(1+r)^i} = \sum_{i=1}^{n} \frac{{}_iP_x Y^*}{(1+r)^i} \tag{2}$$

where X^* is the sum insured, the * superscript denotes an actuarially fair price or amount, ${}_iD_x$ is the probability that a person aged x when the life insurance policy is purchased dies in year i, and $\sum_{i=1}^{n} {}_iD_x = 1$.

Insurance companies sell both life insurance policies and annuities, which has the advantage that the risks of these two types of business tend to be offsetting. For example, underestimating longevity means that annuities are underpriced, while life insurance is overpriced, Cox and Lin (2007). It is assumed that insurance companies use the same longevity estimates when pricing both life insurance and annuities.[6] Therefore, in a competitive environment with no load factors or other frictions, the prices of both annuities and life insurance are driven to the prices in Eqs. (1) and (2), and a fixed relationship exists between

[2] Those who subsequently have lives of above average length are more likely to buy annuities which produces adverse selection against the insurance company. In response insurance companies raise their prices to offset the effects of adverse selection.

[3] The MWR is the present value of the expected annuity payments, divided by the price of the annuity. A MWR of one means that the annuity has an actuarially fair price. MWRs are based on the mortality table chosen by the insurance company, and the set of future interest rates used by the insurance company as discount factors.

[4] Until April 2015 at least 75% of UK defined contribution pension pots had to be used to buy an annuity before the age of 75 (although some exceptions existed) As from April 2015 the 2014 UK budget removed any compulsion for members of defined contribution schemes to buy an annuity.

[5] This is pure life insurance with no savings component.

[6] The UK Continuous Mortality Investigation produces the mortality tables used by insurance companies for pricing both annuities and life insurance. If insurers choose to use unrealistically low forecasts of improvements in longevity for pricing life insurance, they are effectively imposing a load factor on life insurance. Although competitive forces will tend to eliminate such behaviour, to the extent it remains, it can be incorporated in the load factors considered below.