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The role of jump dynamics in the risk–return relationship

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ABSTRACT

Surprisingly, a positive risk–return relationship has not been consistently observed for the traditional GARCH in the mean model in other studies. In this paper, we employ a combination of the jump diffusion and GARCH model in the mean equation to test the risk–return relationship for U.S. stock returns. The results suggest a statistically significant relationship between risk and return if the risk measure includes components of smoothly changing variance and jump events.

1. Introduction

The analysis of the risk–return relationship for equities is a central concern of asset pricing. An important part of the risk–return literature attempts to establish a positive relationship at the market level between the time-varying expected risk premium and its conditional variance over time. Documenting this relationship is a necessary condition for establishing a risk-return relationship at the individual stock level, both cross-sectionally and over time. The GARCH in the mean model (denoted by GARCH-M), introduced by Engle, Lilien, and Robins (1987), allows researchers to jointly estimate both the mean and the variance processes and thereby establish the risk–return connection at the market level.

Researchers who have employed GARCH-M have reported mixed results. For example, employing instrumental variables, Campbell (1987) found a significant negative relationship. In contrast, French, Schwert, and Strambaugh (1987) found a positive relationship between return and conditional variance. Baillie and DeGennaro (1990), Campbell and Hentschel (1992), and Chou (1992), among others, found a positive but insignificant relationship between risk and return. Glosten, Jagannathan, and Runkle (1993) found an inverse relationship between the conditional variance and the expected return. Lanne and Luoto (2008) concluded that, contrary to previous findings, GARCH-M models produced a strong robust relationship between risk and return. However, their results depend on the prior belief about the magnitude of the intercept term. The failure to confirm a risk–return relationship that is positive for the whole market casts suspicion on the research for establishing this relationship for individual stocks.

Scruggs (1998) presents an excellent summary of the conflicting results. Attempting to resolve the enigma by introducing a second factor, long-term government bond returns, he concludes that there is a significant and positive relationship between risk and return. In the same spirit, Wagner and Marsh (2005) introduce what they refer to as "surprise volume" into the GARCH-M model. They also find a positive relationship between return and risk.

One possible reason for the failure to find a positive relationship between risk and return is that the GARCH-M model is only meant to capture smooth changes in the returns and smooth and persistent changes in the variance. As Cont (2001) noted, returns on many financial assets (including stocks) exhibit volatility clustering. That is, volatility has a positive autocorrelation that tends to be high in some periods and low in other periods. Although the GARCH-M model is well suited to explain this phenomenon, it is not well suited to explain large sudden jumps in returns. This is because stock returns are characterized by occasional large, discrete jumps in returns usually due to large shocks commonly attributed to unexpected news announcements. As Boudt, Danielsson, and Laurent (in press) note, applying GARCH models under these circumstances leads to poor estimates of volatility. We believe this may account for the mixed results reported for the GARCH-M model. Thus, incorporating jumps into the GARCH-M model presents another way to measure the risk–return relationship and to shed light on the anomaly. Since jumps cannot be observed directly, it is difficult to estimate them. To make estimation straightforward, Chan and Maheu (2002) proposed a filter to infer ex post the distribution of jumps at time t. Using this filter, they then constructed the shock to the expected

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number of jumps. This shock at time t provides a basis for producing the conditional jump intensity.

Adapting the procedure suggested by Chan and Maheu (2002), in this paper we derive an autoregressive jump intensity (ARJI) model and integrate it into the GARCH-M model. The GARCH model accounts for the smooth and persistent changes in volatility over time, whereas the ARJI model deals with the spikes in returns. The ARJI model not only handles the spikes but also takes into account the clustering of spikes. Moreover, the ARJI model allows for the jump size to vary over time. We believe that combining these models will maintain the GARCH model's ability to account for volatility clustering and the ARJI model's ability to account for sharp jumps in returns. Furthermore, only when the jumps are accounted for can the GARCH-M model detect the risk–return relationship. We refer to this model as an ARJI-GARCH in the mean model (ARJI-GARCH-M model).

This paper verifies the equity risk–return relationship using a measure of risk that includes not only the conditional variance but also jumps. The advantage of the ARJI GARCH-M model compared to the GARCH-M model is that both returns and the conditional variance now include the impact of jumps. Using this model we document a positive relationship between return and risk. Furthermore, we find a positive relationship between return and the risk attributable to jump risk.

Christoffersen, Jacobs, and Ornthanalai (2012) using daily data also adopt this approach to measure the risk premium of dynamic volatility and dynamic jump intensity in both stock index returns and stock index options. Their best performing models for equity returns find that only jump risk is priced. They find no risk premium for the normal dynamic volatility. These results are plausible for short-term investors, such as option traders. We believe it is important to examine whether these results apply to longer-term investors.

In the next section, we outline the model. In Section 3 the data are presented, followed by the reporting of our results in Section 4. In the final section, Section 5, we summarize our findings.

2. ARJI-GARCH-M model

Ball and Torous (1983) provided statistical evidence of the existence of log normally distributed jumps for most NYSE-listed common stocks, suggesting that jumps in stock prices do not have a continuous sample path. Under these conditions, returns are no longer completely described by their mean and variance. The variance as traditionally measured requires the assumption of normally distributed stock returns. The introduction of jumps allows us to account for the non-normality observed in stock returns (see Maheu & McCurdy, 2004; Rachev, Menn, & Fabozzi, 2005). Thus, the risk measure now includes both the traditional variance as well as the jumps.

Furthermore, stock returns not only tend to show jumps but the jumps tend to cluster. For example, the recent financial crises contained episodes of extreme volatility followed by a series of jumps over a relatively short period. As a result, it is important to model both time variation and clustering in the jump process, and for this reason, we adopt the mixed GARCH-jump model of Chan and Maheu (2002). Their model in addition to being well suited for this purpose offers two additional advantages. First, it captures normal news innovations in the conditional variance of returns (GARCH) while unusual news innovations are captured in the time-varying jumps and in the jump intensities (clustering of jumps). Second, the estimation of time-varying jumps and jump intensities requires the identification of news events. The identification of the news events to include in the time-varying jump and the jump-intensity model could be criticized as merely ad hoc.

Chan and Maheu (2002) skirt this problem by introducing a new discrete model in which both the time-varying jumps and jump clustering evolve endogenously according to a parsimonious autoregressive moving average (ARMA) structure. Their model postulates that returns are governed by the simple diffusion model and one additional source of volatility, a Poisson jump model of stock returns. In this model, stock returns experience a discrete number of shocks in each period. The number of shocks or jumps (which could be zero) is determined by a Poisson process. The size of each jump is assumed to be independent and normally distributed with a mean of θ, and a standard deviation of δ. The model for excess stock return R_t, is then:

$$R_t = \mu + \sum_{t=1}^{n} \phi_t R_{t-n} + \sqrt{h_t z_t} + \sum_{k=1}^{n_t} y_{t,k} \quad z_t \sim NI(0,1) \quad y_{t,k} \sim N\left(\theta, \delta^2\right). \tag{1}$$

The jump term, $y_{t,k}$, represents the size of a discrete jump k at time t. The model proposes that n_t, the number of jumps that arrive between $t-1$ and t, follows a Poisson distribution with the Poisson parameter $\lambda > 0$ and the density given by

$$P(n_t = j|\Phi_{t-1}) = \frac{\exp(-\lambda_t)\lambda_t^j}{j!}. \tag{2}$$

The mean and variance for the Poisson random variable are both λ_t, which is called the jump intensity. Chan and Maheu (2002) propose the following approximate ARMA model of the conditional jump intensity

$$\lambda_t = \lambda_0 + \rho\lambda_{t-1} + \gamma\xi_{t-1} \tag{3}$$

where ξ_{t-1} represents the previous period shock in predicting λ_{t-1}. Thus, λ_t depends on its previous value and the jump intensity residual of the previous period. To insure λ_t is positive, we require $\lambda_0 > 0$ and $\rho > \gamma > 0$. Thus, if the previous period λ_{t-1} was higher than expected ($\xi_{t-1} > 0$), then the model predicts that the expected number of jumps will increase. Further, if the appropriate stability conditions are met, then the unconditional value of λ_t is $E(\lambda_t) = \lambda_0/(1-\rho)$.

The parameters of the jump size, $y_{tk,}$ define the jump size distribution. The mean of the distribution is allowed to reflect the asymmetric effects of good and bad news as determined by the sign of the parameter of the risk premium. Each jump has a different size, determined by the normal distribution with mean θ and standard deviation δ. In the jump diffusion model, these size parameters also vary over time. Chan and Maheu (2002) postulate that they depend on past excess returns in the following manner:

$$\theta_t = \eta_0 + \eta_1 R_{t-1} \, D(R_{t-1}) + \eta_2 R_{t-1}(1 - D(R_{t-1})) \tag{4}$$

where $D(R_{t-1}) = 1$ when $R_{t-1} > 0$ and 0 otherwise. This allows the jump size to respond asymmetrically to positive and negative excess returns resulting from good or bad news. If excess returns are positive (negative), then the jump size, $\theta_t = \eta_0 + \eta_1 R_{t-1}$ ($\theta_t = \eta_0 + \eta_2 R_{t-1}$), and the value of η_1 (and η_2) indicate the impact of positive (negative) excess returns. The variance of the jump size depends upon the GARCH variance in the following way:

$$\delta_t^2 = \zeta_0^2 + \zeta_1 \, h_t \tag{5}$$

To test the temporal relationship between risk and return, we extended the GARCH-M model to include the ARJI components in the mean equation. The variance term is no longer the smoothed conditional variance but the conditional variance which contains the following jump model terms:

$$\mathrm{Var}\left(R_t|\Phi_{t-1}\right) = h_t + \left({\delta_t}^2 + {\theta_t}^2\right)\lambda_t. \tag{6}$$