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# GRANTISM: An Excel$^{\text{TM}}$ model for Greenland and Antarctic ice-sheet response to climate changes

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**Abstract**

Over the last decades, the response of large ice sheets on Earth, such as the Greenland and Antarctic ice sheets, to changes in climate has been successfully simulated with large-scale numerical ice-sheet models. Since these models are highly sophisticated, they are only applicable on the scientific level as they demand a large amount of CPU time. Based on similar physics, a computationally fast flowline model of the Greenland and Antarctic ice sheet is presented here, primarily designed for educational purposes. Using an over-implicit numerical scheme, the model runs fast and behaves in a similar way to changes in background temperature forcing as major ice-sheet models do. A user-friendly interface and the implementation within a common spreadsheet program (Excel$^{\text{TM}}$) make the model suitable for the classroom.


*Keywords:* Antarctica; Greenland; Ice-sheet model; Spreadsheet; Climate change

## 1. Introduction

The Antarctic and Greenland ice sheets are the two largest ice masses on Earth. The Antarctic ice sheet covers an area of more than $13 \times 10^6\,\text{km}^2$ and contains more than 80% of the world's fresh-water supply. The Greenland ice sheet—the Northern Hemisphere counterpart of Antarctica—is ten times smaller and has a shorter mass turnover time, so that it has a faster response time to environmental change. Due to the generally long time scales involved, i.e. longer than direct observations can account for, numerical modelling is a vital tool for understanding and predicting the past, current and future behaviour of these ice masses. Nowadays, sophisticated three-dimensional thermomechanical ice-sheet models are capable of explaining and predicting the waxing and waning of the Greenland and Antarctic ice sheet under changing environmental conditions with a relatively high degree of confidence. Such large-scale models are based on continuum modelling and encompass conservation laws of mass, momentum and energy. Examples for such thermomechanical models are Budd and Smith (1982), Huybrechts (1990), Huybrechts (1992), Fastook and Prentice (1994), Hulbe and MacAyeal (1999), Savvin et al. (2000), and Ritz et al. (2001) for the Antarctic and Letreguilly et al. (1991), Huybrechts (1994b), Greve (1997), and Ritz et al. (1997) for the Greenland ice sheet.

Due to the problem complexity and the large amount of input data required, such models need much CPU time to calculate. Here, I present an ice-sheet model at low resolution along a flowline through the Antarctic and Greenland ice sheet, respectively, that

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runs fast even on a modest PC. The model has the same physical basis as the models cited above. Complex interactions are parameterized in such a way that the behaviour of the classroom model is comparable to the findings with the more complex and high-resolution counterparts. However, ice stream behaviour and proper grounding line migration are not accounted for (as is the case in some 3D models), which limits the application of the Antarctic model essentially to an East-Antarctic type of ice sheet instead of a marine ice sheet such as the West-Antarctic ice sheet. Similar simplified models exist in the literature (MacAyeal, 1994; Oerlemans, 2002, 2003). A major difference lies in the use of a common spreadsheet program to perform the calculations, which makes it also easy to use. Furthermore, the numerical over-implicit scheme allows for a fast execution time of the model over time spans that cover several hundred thousands of years. Finally, the model is designed in such a way that it can be employed on different levels. At High-school level it is aimed at investigating the impact of background temperature change on ice sheets (in such case only one parameter has to be changed). At undergraduate level the impact of physical and environmental processes—of which temperature is one of them—on the behaviour of ice sheets can be studied. At graduate level, the model can be applied to other ice masses or glaciers, and adapted to different boundary conditions.

The response of the Antarctic ice sheet to changes in background temperature is likely to be different for the Greenland ice sheet. Because of the low mass turnover—due to the extremely cold surface temperatures, low accumulation rates and large size—the Antarctic ice sheet is considered to remain relatively stable for 100-year time scales under warming scenarios of up to 20 °C (Huybrechts, 1994a). This follows because the increased moisture content in warmer air masses leads to increased precipitation rates. The situation in Greenland is quite different. Surface melting is prevalent during summer around the ice sheet perimeter, but not higher up as temperatures remain year-round below freezing. Surface melting in the ablation zone presently accounts for roughly half of the mass loss from the Greenland ice sheet. The general consensus suggests that modest-to-moderate warming over the next millennium will lead to a gradual, dynamic interplay between marginal mass loss in the ablation zone and increased mass gain in the accumulation zone resulting in a slow, monotonic ice-sheet retreat (Church et al., 2001). This different ice-sheet behaviour for both Greenland and Antarctic ice sheets is clearly demonstrated with the classroom model, even though ice physics are the same for both modeled ice sheets.

## 2. Model description

### 2.1. Field equations

GRANTISM (GReenland and ANTarctic Ice Sheet Model) is a completely dynamic ice-sheet model, based on conservation laws of mass and momentum. For large ice masses such as the Greenland and Antarctic ice sheet, the *shallow-ice approximation* holds, which states that the driving stress $\tau_d$ in an ice mass is balanced by the vertical shearing at the bed $\sigma(b)$, or

$$\sigma(b) \sim \tau_d = -\rho_i g H \nabla h, \tag{1}$$

where $H$ is the ice thickness, $h$ the surface elevation of the ice sheet, $\rho_i$ is the ice density and $g$ the acceleration due to gravity. Stresses are related to strain rates by means of Glen's flow law, from which the vertical mean horizontal velocity in an ice sheet due to internal deformation $\bar{u}_d$ can be derived (Paterson, 1994):

$$\bar{u}_d = \frac{2}{n+2} A(T) H \tau_d^n, \tag{2}$$

where $A(T)$ is a temperature dependent flow parameter, defined by a modified Arrhenius relationship (Hooke, 1981):

$$A(T) = m\left(\frac{1}{B_0}\right)^n \exp\left[\frac{3C}{(T_r - T)^K} - \frac{Q}{RT}\right]. \tag{3}$$

Here, $T$ (K) is the ice temperature, and $m$ is a tuning parameter. Values for this and all other parameters and constants are listed in Tables 1 and 2. The velocity at the base of the ice sheet is given by

$$u(b) = A_b \frac{\tau_d^p}{Z^\star}, \tag{4}$$

where $p = 3$ is a sliding law exponent, and $Z^\star$ is the height of the ice surface above buoyancy level, defined

Table 1
List of constants and global parameters

| Constant | Value | Units | Definition |
|---|---|---|---|
| $\rho_i$ | 910 | $\mathrm{kg\,m^{-3}}$ | Ice density |
| $\rho_s$ | 1028 | $\mathrm{kg\,m^{-3}}$ | Sea-water density |
| $\rho_m$ | 3300 | $\mathrm{kg\,m^{-3}}$ | Mantle density |
| $\theta$ | 3000 | a | Relaxation time asthenosphere |
| $B_0$ | 2.207 | $\mathrm{Pa\ a^{1/n}}$ | |
| $C$ | 0.16612 | $\mathrm{K}^K$ | |
| $K$ | 1.17 | | |
| $n$ | 3 | | Flow law exponent |
| $p$ | 3 | | Sliding law exponent |
| $Q$ | $7.88 \times 10^4$ | $\mathrm{J\,mol^{-1}}$ | Activation energy for creep |
| $R$ | 8.31 | $\mathrm{J\,mol^{-1}\,K^{-1}}$ | Universal gas constant |
| $T_r$ | 273.39 | K | |