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Simple unit root testing in generally trending data with an application to precious metal prices in Asia

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ABSTRACT

This paper proposes a new unit root test that is general enough to accommodate a potentially non-linear deterministic trend function, making it one of the most general tests around. However, the main advantage lies with its simple implementation. In particular, the asymptotic critical values are shown to be "almost" independent of the deterministic trend function, and as a result the test can be implemented without the need for model-specific critical values. The new test is applied to a sample consisting of monthly prices of four precious metals for a number of Asian countries.

1. Introduction

One of the main problems in practice when testing for the presence of a unit root is that the stochastic part of the series cannot be observed directly, but is instead observed subject to some additive deterministic component of unknown form. Valid inference on the unit root hypothesis therefore relies critically on the researcher being able to account for the confounding effects of that component. When working with stationary time series, this is relatively simple; all one has to do is to augment the model of interest with whatever deterministic specification felt appropriate (usually after plotting the data), which, if general enough to nest the true model, makes inference invariant with respect to the unknown deterministic component. Unfortunately, when working under the unit root matters are more complicated, as in this case the asymptotic distribution, and hence also the critical values, of the unit root test depend on the chosen deterministic specification (which need not be equal to the true one). In finance, taking a constant term as given, the question is often whether or not to include a linear time trend, and for these default specifications (constant or a constant and trend) there are critical values available (see, for example, MacKinnon, 1996).

Of course, for many financial time series a linear trend, rather than just a constant, is probably more appropriate as the default specification. This would be case for series such as stock prices, money, income, exchange rates and interest rates,

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where trending behavior is evident (see Harvey, Leybourne, & Taylor, 2011). For instance, Murray and Nelson (2000) argue that a linear trend for real post-Word War II GDP for the US at times produces implausible departures from that trend. To take one example, their trend stationary model implies that real GDP was 8% below trend in 1997, yet unemployment was simultaneously very low.

The question therefore arises as to whether the linear trend is sufficiently general to capture actual behavior or whether some more flexible specification for the trend function should be considered, suggesting that the default critical values with (at most) a linear trend are generally not enough. One of the most popular approaches to relax the linear trend assumption is to allow for structural change in level or trend, and in so doing it is common to assume a pre-specified number of breaks, usually one or two (see Lee & Strazicich, 2003). Of course, this does not mean that there cannot be more than two breaks (see, for example, Murray & Nelson, 2004, in the case of real GDP); it just means that currently suitable unit root tests are not readily available. In particular, the problem is that the asymptotic test distribution in this case depend on both the number of breaks and their location, suggesting that critical values have to be tabulated for every possible combination of breakpoints, which is practically infeasible for more than two breaks. Then there is also the assumption that the breaks have to be equally spaced, as otherwise critical values for a continuum of breakpoints would be required. Thus, while the modeling of the breaks is an easy task (using dummy variables), if suitable critical values are not available this means that the test too is unavailable. A similar problem occurs when using polynomial trends or trigonometric functions to approximate smooth breaks, but then the dependence is instead on the trend degree and frequency of the approximation, respectively (see Enders & Lee, in press; Ouliaris, Park, & Phillips, 1989).

The above mentioned issue would be less problematic had there been something in the way of consensus as to which approach is the most appropriate. Unfortunately, this is not the case, which is why applied researchers tend to accumulate huge buffers of critical values (and tests) to fit every possible need. In this paper we take this as our starting point. The aim is to device a test for a unit root that is general enough to accommodate most relevant deterministic models around, but that does not rely to such an extent on having the correct critical values. The approach taken builds on the work of Shin and So (2001), showing how the use of recursive demeaning in a model with only a constant has a bias-reducing effect on the estimation of the autoregressive coefficient. Of course, our aim is quite different; however, intuition suggests that recursive demeaning should have implications also for the extent to which the asymptotic distribution of unit root tests depend on the chosen deterministic specification. Our results confirm this.

Ideally the asymptotic test distribution would be completely independent of the deterministic specification. While the recursive detrending-based, or REC-based, test statistic considered in this paper does not go all the way, it is very close. Specifically, although the shape of the asymptotic distribution does depend on the deterministics, the left tail critical values are "almost" constant. In order to get a feeling for the accuracy of the constant approximation, the worst distortions we find when using the 5% critical value corresponding to a third-order trend polynomial but the test regression is fitted with just a constant is 3.3%. Thus, if this degree of size distortion is acceptable, then there is no need to keep with model-specific critical values, which, as already alluded, is a huge advantage in practice. Of course, there is still the issue of how to pick the appropriate critical value to use, and in the paper we provide some recommendations toward this end.

The plan of the rest of the paper is as follows. Section 2 sets out the model and assumptions, which are used in Section 3 to establish the asymptotic properties of the REC unit root test statistic. The small-sample performance of the new test is investigated by means of Monte Carlo simulation in Section 4. The implementation of the new test is illustrated in Section 5 using as an example the prices of four precious metals. The sample stretches the period January 2000 to June 2012, and includes 9–11 Asian countries for each metal. Section 6 concludes. Proofs of the main theoretical results of this paper are given in an appendix.

2. Model and assumptions

The data generating process (DGP) of the observed variable $\{y_t\}_{t=1}^{T}$ is given by

$$y_t = \beta' D_t + u_t, \tag{1}$$

$$u_t = \rho u_{t-1} + v_t, \tag{2}$$

$$\phi(L) v_t = \epsilon_t, \tag{3}$$

where D_t is an q-dimensional vector of trend functions with a one as its first element, ϵ_t is iid with $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma^2 > 0$, $E(\epsilon_t^4) < \infty$, $u_0 = 0$, and $\phi(L) = 1 - \sum_{k=1}^{p} \phi_k L^k$ is a pth order polynomial in the lag operator L that has all its roots outside the unit circle. If $p = 0$, then we define $\phi(L) = 1$. Also, there exist a $q \times q$ diagonal normalization matrix J_T such that $J_T d_t \to d(u)$ as $T \to \infty$, where $u \in [0, 1]$, $\int_{u=0}^{w} d(u) d(u)' du$ is positive definite for all $w \in (0, 1]$, $d_t = H \Delta D_t$ and H is a $q \times q$ selection matrix of zeros and ones that removes the first row of ΔD_t (which equals zero).

The autoregressive coefficient, ρ, is assumed to be local-to-unity in the following sense:

$$\rho_t = \exp\left(\frac{c}{T}\right), \tag{4}$$