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Pricing and disentanglement of American puts in the hyper-exponential jump-diffusion model[☆]

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We analyze American put options in a hyper-exponential jump-diffusion model. Our contribution is threefold. Firstly, by following a maturity randomization approach, we solve the partial integro-differential equation and obtain a tight lower bound for the American option price. Secondly, our method allows to disentangle the contributions of jumps and diffusion for the early exercise premium. Finally, using American-style options on the S&P 100 index from January 2007 until December 2012, we estimate various hyper-exponential specifications and investigate the implications for option pricing and jump-diffusion disentanglement. We find that jump risk accounts for a large part of the early exercise premium.

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1. Introduction

The valuation of American options has been one of the most important topics in mathematical finance for almost five decades. A fully analytic solution to the problem, even in the simplest setting, has not yet been obtained whatsoever. The main difficulty stems from the fact that American options allow for an early exercise feature, which requires solving for the option price as a function of a free boundary that is not known a priori. A common approach is to use numerical procedures.[1] However, numerical methods are generally devoid of financial intuition and meaningful interpretations. Moreover, their implementation is often computationally expensive, even more so when we leave the classical Black and Scholes (1973) setting.

A huge part of the literature on analytic pricing of American options in non-Gaussian settings deals with perpetual American options, which can be solved in closed-form under certain assumptions regarding the jumps of the underlying process, and because the early exercise boundary is flat.[2] Moreover, they very often exclude the possibility of overshooting a predefined constant barrier by confining jumps to be always in the opposite direction from the barrier. Hence, many models are based on spectrally one-sided Lévy processes in order to utilize renewal-type integral equations or fluctuation identities. The possibility of an overshoot of the exercise boundary poses several mathematical problems. We need not only an exact distribution of the overshoot, but also the dependency structure between the overshoot and the first passage time. Moreover, for finite-maturity American options, the optimal stopping time is actually a first passage time of an unknown non-uniform space-time boundary.

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[1] Broadie and Detemple (2004) and Detemple (2005) provide a comprehensive overview of different pricing methods for American-style options.

[2] See, e.g., Boyarchenko and Levendorskiĭ, 2002; Mordecki, 2002; Chesney and Jeanblanc, 2004; Alili and Kyprianou, 2005 and references therein.

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Our first goal in this paper is the valuation of finite-maturity American put options in the hyper-exponential jump-diffusion model (HEJD) introduced by Lipton (2002). The logarithm of the asset price is assumed to follow a process, which is a mixture of a drifted Brownian motion and a compound Poisson process with an arbitrary number of positive and negative types of exponentially distributed jumps of finite activity.[3] We choose to work in this particular framework, because it is flexible enough to capture the main empirical features of asset returns and option prices.[4] Furthermore, due to the memorylessness of the exponential distribution, analytic pricing and hedging of vanilla and certain exotic options in a HEJD framework is feasible, hence making this model a plausible candidate for our purpose.

To derive the price of an American put, we adopt a maturity randomization approach. We study the Laplace-Carson transform with respect to the time to maturity of the partial integro-differentialequation (PIDE) and the corresponding initial and boundary conditions describing the dynamics of the American option price. Instead of using a sequence of Erlangian random variables suggested in Carr (1998), we rely on a different sequence of random variables following a distribution suggested in queueing theory literature by Gaver (1966). Both approaches converge pointwise to Dirac's delta function centered at the residual maturity. However, while Carr's maturity randomization requires to solve recursively a set of differential equations, the alternative randomization approach relies on the computation of the Gaver's functionals, resulting in a much simpler and faster computational procedure. For option pricing applications, the alternative randomization approach was studied in, e.g., Kou and Wang (2003); Sepp (2004); Kimura (2010), and Hofer and Mayer (2013). However, American options have not yet been priced using this method. Hence, our results represent a genuine contribution to the existing literature on maturity randomization.[5] We support our theoretical results with numerical examples and demonstrate that our approach represents a fast and accurate pricing engine.

Our second contribution is concerned with analyzing the importance of jump risk for American options. Although we borrow the syntax "disentanglement of diffusion from jumps" from Aït-Sahalia (2004), the semantics is quite different in our study. While Aït-Sahalia (2004) takes an econometric perspective and studies the effect of jumps on the estimation of the diffusion component in asset returns using high-frequency data, we study the implications of a possible overshoot for the early exercise premium. By combining the martingale approach and the PIDE method in the Laplace transform framework, we can disentangle the contribution of jumps and diffusion for the American early exercise premium. Our disentanglement result pertains to the risk-neutral world and is model-dependent in the sense that it relies on the assumption that the underlying process has both diffusion and finite activity jump components. However, unlike econometric approaches, it holds irrespectively of the data frequency.

The impact of jumps on American option prices has been recently considered in Chiarella and Ziogas (2009). They examine jump effects by comparing the shape of the early exercise boundary with and without jumps, keeping the overall volatility constant. In contrast, we consider the disentanglement of jumps and diffusion directly by analytically decomposing the early exercise premium into the respective contributions. Hence, we do not need to rely on a moment-based condition. More importantly, our approach does not require the use of different models, which may distort the inference due to model misspecification. In our setting, we can disentangle jumps from the diffusion component within the same model. Therefore, to the best of our knowledge, our disentanglement idea for American options has not been previously studied in the literature.

Finally, as our third contribution, we estimate a range of HEJD models using American options on the S&P 100 index to provide more intuition and to demonstrate the disentanglement of jumps from diffusion on real data. We focus on short-term options with time to maturity of up to two months and perform sequential (weekly) calibration via penalized weighted nonlinear least squares. The chosen data set and the sequential calibration allow us to study the time variation in the model parameters. Since our sample includes the recent financial crisis, we can study the importance of diffusion and jumps during calm and turbulent periods.

The reminder of the paper is organized as follows. Section 2 introduces the hyper-exponential jump-diffusion model and presents our theoretical contributions regarding the pricing of European and American put options, and the early exercise jump-diffusion disentanglement. In Section 3, we present and discuss our calibration results as well as the implications for the disentanglement. Section 4 concludes the paper.

2. Option pricing and disentanglement in the HEJD model

2.1. Model formulation

We consider a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = \{\mathcal{F}_t, t \geq 0\}, \mathbb{Q})$ satisfying the usual assumptions. The asset price dynamics under the fixed risk-neutral probability measure $\mathbb{Q}$ follows a hyper-exponential jump-diffusion process

$$\frac{dS_t}{S_{t-}} = (r - \delta - \lambda\zeta)dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t}(V_i - 1)\right), \tag{1}$$

where $\{W_t,\ t \geq 0\}$ is a standard Brownian motion under $\mathbb{Q}$. The interest rate $r \in \mathbb{R}_+$, the dividend yield $\delta \in \mathbb{R}_+$, and the volatility $\sigma \in \mathbb{R}_+$ are constants.[6] The Poisson process $\{N_t,\ t \geq 0\}$ is characterized by the jump intensity parameter $\lambda \in \mathbb{R}_+$ and $\{Y_i := \log(V_i) : i = 1, 2, \ldots\}$ is a sequence of independent and identically distributed hyper-exponential random variables with probability density function

$$\varphi_Y(y) = \sum_{i=1}^{m} p_i \eta_i e^{-\eta_i y} \mathbb{1}_{\{y \geq 0\}} + \sum_{j=1}^{n} q_j \theta_j e^{\theta_j y} \mathbb{1}_{\{y < 0\}}, \tag{2}$$

where $m, n \in \mathbb{N}$. The coefficients $p_i > 0$ for all $i = 1, \ldots, m$ and $q_j > 0$ for all $j = 1, \ldots, n$ are probabilities of different kinds of positive and negative jumps, respectively, satisfying $\sum_{i=1}^{m} p_i + \sum_{j=1}^{n} q_j = 1$. Similarly, the parameters $\eta_i > 1$ for all $i = 1, \ldots, m$ and $\theta_j > 0$ for all $j = 1, \ldots, n$ are magnitude parameters of different kinds of random upward and downward jumps, respectively. Furthermore,

[3] General properties of the HEJD model are thoroughly analyzed and discussed in Cai (2009).

[4] Jeannin and Pistorius (2010) and Crosby et al. (2010) show that any Lévy process with completely monotone Lévy density can be approximated by a HEJD process. Such Lévy models include, e.g., the Variance-Gamma, the Normal-Inverse-Gaussian, the Carr-Geman-Madan-Yor model.

[5] American option pricing in double-exponential and hyper-exponential jump-diffusion setting has been studied Kou and Wang (2004) and Cai and Sun (2014). These papers are based on the quadratic approximation of Barone-Adesi and Whaley (1987). Avram et al. (2002) consider fluctuation theory approach in a spectrally one-sided (positive or negative) Lévy model. Levendorskiĭ (2004a); 2004b) analyze regular Lévy processes of exponential type using the Wiener-Hopf factorization formula embedded in the dynamic programming approach.

[6] We remark that the assumption of positive interest rates is, under reasonable parameter specifications, not restrictive for our analysis and implementation using the Gaver–Stehfest canadization method. However, for this numerical method, we require dividends to be continuous and deterministic.

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