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One-sided performance measures under Gram-Charlier distributions

Angel León[a,1], Manuel Moreno[b,2,*]

[a] Department of Quantitative Methods and Economic Theory, University of Alicante, San Vicente del Raspeig, Alicante 03080, Spain
[b] Department of Economics and Finance, Manuel Moreno is from University of Castilla La-Mancha,Toledo 45071, Spain

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ABSTRACT

We derive closed-form expressions for risk measures based on partial moments by assuming the Gram-Charlier (GC) density for stock returns. As a result, the lower partial moment (LPM) measures can be expressed as linear functions on both skewness and excess kurtosis. Under this framework, we study the behavior of portfolio rankings with performance measures based on partial moments, that is, both Farinelli-Tibiletti (FT) and Kappa ratios. Contrary to previous results, significant differences are found in ranking portfolios between the Sharpe ratio and the FT family. We also obtain closed-form expressions for LPMs under the semi non-parametric (SNP) distribution which allows higher flexibility than the GC distribution.

1. Introduction

An adequate risk-adjusted return performance measure (PM) is essential for selecting investment funds. The Sharpe ratio (Sharpe, 1966; 1994) has become the bechmark PM by adjusting the expected excess fund return by the symmetric risk measure or standard deviation. Although this ratio is still a reference indicator for assessing the accuracy of investment strategies, its use becomes rather doubtful when the fund return distribution is beyond the class of elliptical distributions (Owen and Rabinovitch, 1983) that include the normal distribution. As a result, several one-sided type measures of risk have been proposed and the associated PMs are known as one-sided PMs. In fact, some of these PMs are also characterized by one-sided reward measures.

Some examples of one-sided PMs are the adjusted for skewness Sharpe ratio (*ASSR*) proposed by Zakamouline and Koekebakker (2009a), the Generalized Rachev family based on the conditional Value at Risk (Biglova et al., 2004), the Farinelli-Tibiletti (FT) family based on both upper and lower partial moments (Farinelli and Tibiletti, 2008) and the Kappa or S-S family (Sortino and Satchell, 2001) based on lower partial moments. Other alternative reward-to-variability ratios are well documented in Caporin et al. (2014) and the references therein. We will also implement PMs based on the certainty equivalent amount as a function of both prudence and temperance coefficients. These coefficients are related to the investor's appetite for asymmetry and aversion to leptokurtocity of fund returns. For details, see Ebert (2013); Eeckhoudt and Schlesinger (2006) and references therein.

Some very interesting papers as Eling (2008); Eling and Schuhmacher (2007), and Auer (2015) find that choosing different PMs is not critical to the portfolio evaluation. More specifically, the PM choice does not matter because any PM generates the same rank ordering as the Sharpe ratio (*SR*). Guo and Xiao (2016) agree with this result whenever the selected PMs satisfy the monotonic increasing property regarding the *SR* when the fund return distributions belong to the location-scale (LS) family.[3]

In contrast, our paper shows that some PMs like the FT family can generate different rank scores meaning that the choice of the PM is a relevant issue. This is in line with some empirical evidence such as the US mutual fund ranking indicated in Haas et al. (2012), where a member of the FT family (the Upside Potential ratio) exhibits lower Spearman's correlations (with respect to the *SR*)

* Corresponding author.
E-mail addresses: aleon@ua.es (A. León), manuel.moreno@uclm.es (M. Moreno).
[1] Fax: (+34) 965903621.
[2] Fax: (+34) 925268801.

[3] See also Schuhmacher and Eling (2011); 2012). Some probability density functions that satisfy the LS condition are: Beta, Cauchy, Exponential, Extreme Value, Gamma, Logistic, Normal, *t*-Student, Uniform, Weibull, and Normal Inverse Gaussian (under some parametrizations). On the contrary, the lognormal density does not verify this condition.

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than other PMs. Our results also agree with the evidence shown in Cogneau and Hübner (2009) and Zakamouline (2011).

We get closed-form expressions for the FT measures by assuming a return distribution that does not belong to the LS family. To be more precise, we consider the Gram-Charlier (GC) expansion as the probability density function (pdf). The GC distribution has been implemented, among others, by Corrado and Su (1996); Jondeau and Rockinger (2001) and Jurczenko and Maillet (2006). The advantage of this distribution is that both skewness (s) and excess kurtosis (ek) appear directly as the pdf's parameters. We previously get the closed-form expressions for the lower partial moment (LPM) measures as simple linear functions of both parameters. As a consequence, we can easily understand the behavior of these risk measures regarding changes in these higher moments. By expressing the upper partial moment (UPM) measures in terms of LPMs, we can focus just on this kind of downside risk measure[4] and analyze its properties under the GC distribution when studying the FT measures. Similarly, we obtain closed-form expressions for the Kappa measures under the previous distribution.[5]

Finally, the GC restriction to capture higher levels of s and ek suggests some other candidate distributions to seize better these higher moments but, unfortunately, leading to more complex expressions for the PMs. For instance, a more flexible distribution to the restricted higher moments under the GC distribution can be the semi-nonparametric (SNP) density proposed by Gallant and Nychka (1987). We also obtain the LPM analytical expressions under the SNP distribution.

The rest of the paper is organized as follows. In Section 2 we present different PMs based on either the Expected Utility Theory (EUT) or the Prospect Theory/Cumulative Prospect Theory (PT/CPT), see Kahneman and Tversky (1979) and Tversky and Kahneman (1992). Section 3 shows the GC distribution and some properties. In Section 4 we obtain closed-form expressions for LPM and UPM measures under GC and hence, the expressions for both FT and Kappa ratios. We also analyze the behavior of the Kappa ratios regarding the levels of s and ek and obtain the isocurves for the Kappa measures. In Section 5 we conduct a simulation study on the performance evaluation. Section 6 shows the SNP distribution and the corresponding LPM expressions. Finally, Section 7 summarizes and provides the main conclusions. The proofs of propositions and corollaries are deferred to a final technical Appendix.

2. Performance measures (PMs)

Let $U(W)$ denote the investor's utility function where W is the amount of wealth. The investor faces a capital allocation problema that is solved by maximizing his expected utility of wealth $E[U(W)]$, where $E[\ \cdot\ ]$ is the expectation operator. The market includes a risky asset and a risk-free one. Assume that the initial wealth is W_I and the capital allocation aims to invest an amount a in the risky asset and, hence, $W_I - a$ in the risk-free asset. Thus, the investor's final wealth is

$$W(r,a) = a\left(r - r_f\right) + W_I\left(1 + r_f\right), \tag{1}$$

where r is a random variable that denotes the return of the risky asset and r_f is the risk-free rate of return that is assumed to be a constant. Assuming that $a \geq 0$ (short-selling is not allowed), the investor's objective is selecting a to maximize the expected utility:

$$E[U(W(r,a^*))] = \max_a E[U(W(r,a))], \tag{2}$$

where a^* denotes the optimal amount invested in the risky asset from the maximization of the expected utility on the final wealth in (1). Besides EUT as the benchmark model of choice under uncertainty, we are interested in those models under PT/CPT where the utility function is defined over gains and losses relative to some reference point (kink), as opposed to wealth in EUT.

By using the *maximum principle* method,[6] we can rewrite (2) as $E[U(W(r,a^*))] = h(\pi(r))$ where $h(\cdot)$ is a strictly increasing function and $\pi(r)$ represents the PM.[7] More specifically, the investor prefers the risky portfolio r_1 to the risky portfolio r_2 if $\pi(r_1) > \pi(r_2)$. Hence, the aim at maximizing the investor's expected utility can alternatively be formulated as the maximization of a particular PM. In addition, a rational utility-based PM must be consistent with the stochastic-dominance principles that will be analyzed later.

Finally, a GC probability distribution for the returns of the risky asset will be assumed to obtain closed-form PM expressions under both EUT and PT/CPT. The reason for this specific distribution is because we can get a very easy interpretation in terms of the implied distribution parameters which are both skewness and kurtosis.

2.1. PMs based on EUT

These PMs will be obtained by implementing the maximum principle method and using the certainty equivalent (CE) amount corresponding to $E[U(W(r,\ a^*))]$ in (2) for ranking portfolios. Thus,

$$E[U(W(r,a^*))] = \max_a U(CE), \tag{3}$$

such that $CE = \mu_W - \xi$, where $\mu_W = E[W(r,a)]$ represents the expected final wealth and ξ denotes the risk premium.

2.1.1. Certainty equivalent as PM

Let U be a utility function with desirable properties, that is, $U^{(1)} > 0, U^{(2)} < 0, U^{(3)} > 0$ and $U^{(4)} < 0$, where $U^{(i)}$ denotes the i-th derivative of the utility function. We start from the equation defining the CE amount given by

$$U(E[W(r,a)] - \xi) = E[U(W(r,a))]. \tag{4}$$

First, on the right-hand side in (4), approximate the utility function $U(x)$ by a fourth-order Taylor expansion around the point $x_0 = \mu_W$, where $x = W$, and take expectations. Second, on the left-hand side in (4), apply a first-order Taylor expansion around the same point x_0 but now $x = \mu_W - \xi$. Then, the maximum CE amount satisfying (3) is given by

$$CE^* \simeq \max_a \left\{ W_0 + a\left(\mu - r_f\right) - \frac{1}{2}\gamma\sigma^2 a^2 + \frac{1}{3!}s\psi_3\sigma^3 a^3 - \frac{1}{4!}k\psi_4\sigma^4 a^4 \right\}, \tag{5}$$

where μ, σ, s, and k denote, respectively, the mean, standard deviation, skewness and kurtosis of the risky asset return and $W_0 = W_I\left(1 + r_f\right)$. Let $\omega_k = U^{(k)}(\mu_W)$, then $\gamma = -\omega_2/\omega_1$ is the traditional absolute risk aversion coefficient, $\psi_3 = \omega_3/\omega_1$ is the coefficient of *appetite toward asymmetry* and $\psi_4 = -\omega_4/\omega_1$ is the coefficient of

[4] Some seminal references on LPMs are Bawa (1975); Bawa and Lindenberg (1977); Fishburn (1977); Harlow and Rao (1989); Holthausen (1981) and Harlow (1991).

[5] In the same spirit, Passow (2005) obtains a closed-form expression for the Sharpe–Omega ratio (that belongs to the Kappa family) under the (more flexible) Johnson distribution family. The drawback is that the above ratio becomes more cumbersome and, then, more difficult to interpret than when assuming a GC distribution.

[6] This method is presented by Pedersen and Satchell (2002), and later in Zakamouline and Koekebakker (2009a); 2009b) and Zakamouline (2014). Finally, under some conditions, we can find an explicit solution for the amount a.

[7] Note that the PM is not unique since any positive increasing transformation of a PM leads to an equivalent PM.