



Pricing and hedging of derivatives in contagious markets



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ARTICLE INFO

Article history:

Received 30 July 2014

Accepted 31 January 2016

Available online 9 February 2016

JEL classification:

G01

G13

Keywords:

Multivariate modeling

Contagion

Derivatives pricing

Jump processes

Hedging

ABSTRACT

It is well documented that stock markets are contagious. A negative shock to one market increases the probability of adverse shocks to other markets. We model this contagion effect by including mutually exciting jump processes in the dynamics of the indexes' log-returns. On top of this we add a stochastic volatility component to the dynamics. It is important to take the contagion effect into account if derivatives written on a basket of assets are to be priced or hedged. Due to the affine model specification the joint characteristic function of the log-returns is known analytically, and for two specifications we detail how the model can be calibrated efficiently to option prices. In total we calibrate over an extended period of time the specifications to options data on four US stock indexes, and show how the models achieve satisfactory pricing errors. We study the effect of contagion on multi-asset derivatives prices and show how for certain derivatives the impact is heavy. Moreover, we derive hedge ratios for European put and call options and perform a numerical experiment, which illustrates the impact of contagious jumps on option prices and hedge ratios. Mutually exciting processes have been analyzed for multivariate intensity modeling for the purpose of credit derivatives pricing, but have not been used for pricing/hedging options on equity indexes.

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1. Introduction

It is well documented that stock markets are contagious. Markets are heavily interlinked, and losses spread across them. Moreover, financial contagion is magnified as fears of adverse market shocks tend to spread among investors, resulting in panic behavior. Hence, a negative shock to one market increases the probability of negative shocks to other markets, see [Ait-Sahalia et al. \(2015\)](#) and references therein for empirical documentation. We model this contagion effect by including mutually exciting jump processes in the dynamics of the indexes' log-returns, so that a jump in one market increases the intensities of more jumps in the same market and in other markets. Between jumps, the intensities revert to their long-run means. Clustering of jumps can also be achieved by for instance, letting the jump process be specified as a doubly stochastic Poisson process with stochastic intensity. However, this approach will not capture the direct feedback effect that jumps have on the probability of more jumps occurring. On top of this, we add a stochastic volatility component to the dynamics in order to introduce more flexibility in the return distribution. In a recent empirical study, [Ait-Sahalia et al. \(2015\)](#) show that a model of this

type captures the observed joint evolution of equity index returns from different regions of the world.

[Fig. 1](#) depicts an example of successive negative log-returns of the indexes Amex Biotechnology Index (BTK), the Morgan Stanley Technology index (MSH), the Securities Broker/Dealer index (XBD), and the Natural Gas index (XNG) over the period from Tuesday, November 4 to Monday, November 10, 2008. On Tuesday the XBD index and to a lesser degree the MSH index experienced a big drop in value, which was followed by additional drops in both indexes together with the XNG index on Wednesday. On Thursday all four indexes rebounded a bit with returns around 2%. Then on Friday the XBD index fell more than 8%, followed by significant decreases in all four indexes in the beginning of the next week. The model proposed in this paper is well-suited for modeling occurrences like this.

Multi-asset derivatives such as spread and correlation options are traded frequently over-the-counter and on exchanges, and it is important to take the contagion effect into account if they are to be priced or hedged ([Fry-McKibbin et al., 2014](#)). Structured products containing basket options are both traded on exchanges ([Wallmeier and Diethelm, 2012](#)) and also marketed to private investors ([Jørgensen et al., 2011](#)).

Multivariate jump and stochastic volatility models have been proposed in the academic literature, in e.g. [Leoni and Schoutens \(2008\)](#),

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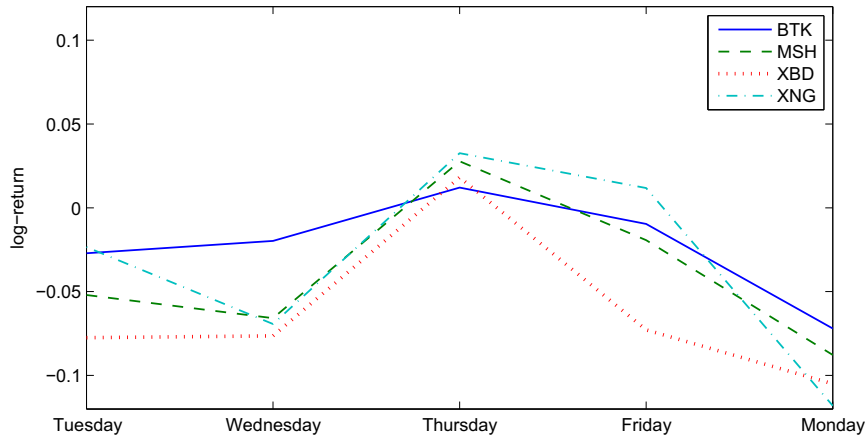


Fig. 1. Historical log-returns of the four indexes: BTK, MSH, XBD, and XNG over the week from November 4–10, 2008.

Gouriéroux et al. (2009), Luciano and Semeraro (2010), and Madan (2011) etc., but none of these are able to model contagious links between different indexes. Self-exciting jump processes were first introduced by Hawkes (1971) and have therefore also been given the name *Hawkes processes*. Hawkes processes have been applied in a number of different contexts, e.g. to model contagious diseases in epidemiology, earthquake occurrences, and in genome analysis etc. (see footnote 4 in Ait-Sahalia et al. (2015) for references). In a financial context, mutually exciting processes have been analyzed for multivariate intensity modeling for the purpose of credit modeling in Errais et al., 2010; Ait-Sahalia et al., 2014, for optimal portfolio selection in Ait-Sahalia and Hurd (2015), and in market microstructure to model transaction times and price changes jointly at high frequency in Bowsher (2007), but have not been used for pricing/hedging options on equity indexes.

Due to the affine model specification, the joint characteristic function of the log-returns is known analytically (Duffie et al., 2000), and for two specifications we detail how the model can be calibrated efficiently to option prices using Fourier transform methods. In total, we calibrate the specifications to single name NASDAQ options data, across strikes and maturities, for the four sector indexes BTK, the MSH, the XBD, and the XNG and investigate their pricing performances over an extended period of time. Preferably, one should also add multi-asset derivatives to the calibration instruments, when calibrating a multivariate model, but due to lack of data we have not been able to do this. In order to reduce computational time, the calibration is performed in two steps. First, the parameters of the mutually exciting processes are calibrated for the indexes simultaneously to a subset of the options data. Second, the stochastic volatility dynamics are calibrated to the whole option surface on each index independently. In general, we find that the two considered multivariate contagion specifications are able to fit the four options markets comparable to the individually calibrated Heston dynamics in most of the maturity/strike spectra of the implied volatility surface.

Moreover, we study the impact of the contagion on the prices of multi-asset derivatives such as spread, correlation and basket options, and find it to have a heavy impact on correlation and basket option prices, while the spread option is less sensitive to the level of contagion.

Finally, we derive the model hedge ratios for European put and call options and study their historical performances in the hedging of European single-name options. Additionally, we perform a numerical study where we analyze the impact of the contagion on option prices and hedge ratios.

The rest of the paper is structured as follows: Section 2 defines an affine jump-diffusive process, introduces the model specifications and derives the characteristic function of the log-return processes. Furthermore, we consider two parametric examples. In Section 3 the two specifications are implemented on options data. Section 4 summarizes how knowledge of the characteristic function for the joint return process can be utilized for the pricing of spread and correlation options using Fourier transform methods, and the impact of the contagion parameters on multivariate option prices is studied. The hedging of European options are studied in Section 5 and finally Section 6 concludes.

2. Contagion modeling

Consider a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{Q})$, where $\{\mathcal{F}_t\}_{t \geq 0}$ is a right-continuous and complete filtration representing the history of the arbitrage-free markets. Assume that the m asset processes S^1, \dots, S^m are adapted to $\{\mathcal{F}_t\}_{t \geq 0}$ and that \mathbb{Q} is a pricing measure. We will assume the instantaneous interest rate r_t and dividend yields q_t^i 's deterministic. We start this section by describing the general affine jump-diffusive setup before introducing the model specifications of the paper.

2.1. Affine jump-diffusions

Since the affine jump-diffusion model framework embeds all the models studied in this paper we start by detailing it here. A jump process is called affine if its arrival intensity is an affine function of an affine jump-diffusion with jump sizes drawn from a fixed distribution (see Duffie et al., 2000). A d -dimensioned Markovian process Y is an affine jump-diffusion if it solves

$$dY_t = \mu(Y_t, t)dt + \sigma(Y_t, t)dW_t + \sum_{i=1}^m \zeta^i dj_t^i, \quad (1)$$

where W is a d -dimensional Wiener process and each J^i is a d -dimensional pure jump process whose component processes jump simultaneously with common intensity λ^i and fixed jump size distribution \underline{E}^i . For $i = 1, \dots, m$, $\zeta^i \in \mathbb{R}^{d \times d}$ the coefficients have the form

$$\begin{aligned} \mu(y, t) &= K_0(t) + K_1(t)y, \quad K_0(t) \in \mathbb{R}^d, \quad K_1(t) \in \mathbb{R}^{d \times d} \\ (\sigma(y, t)\sigma(y, t)^\top)_{jk} &= (H_0)_{jk}(t) + (H_1)_{jk}(t) \cdot y, \quad H_0(t) \in \mathbb{R}^{d \times d}, \quad H_1(t) \in \mathbb{R}^{d \times d \times d} \\ \lambda^i(y, t) &= \Lambda_0^i(t) + \Lambda_1^i(t) \cdot y, \quad \Lambda_0^i \in \mathbb{R}, \quad \Lambda_1^i(t) \in \mathbb{R}^d, \quad i = 1, \dots, m, \end{aligned}$$

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