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Multi-factor volatility and stock returns

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ABSTRACT

In light of inconclusive evidence on the relation between market volatility and stock returns, this paper proposes a multi-factor volatility model and examines its impact on cross-sectional pricing. We also evaluate the out-of-sample performance and economic significance of multi-factor volatility. We find that conditional variances of the size and value dynamic factor earn significant and positive variance risk premia. In addition, multi-factor volatility can significantly improve the out-of-sample return predictability with a positive economic gain in asset allocation.

1. Introduction

The question of whether volatility affects stock returns is an enduring one in financial economics. Merton's (1973) ICAPM interprets the change in market volatility as the time-varying investment opportunity that characterizes a shift in the trade-off between risk and return. In equilibrium, investors taking on additional risk should be compensated through higher expected return, which implies a positive correlation in the volatility-return relationship. However, empirical evidence on the volatility-return relationship is still inconclusive within the single-factor volatility framework.[1] The assumption of a single market volatility may lead to model misspecification if there exist additional sources of volatility risk that characterize a multi-dimensional change in the investment opportunity set. In this paper, we examine the effect of multi-factor volatility on cross-sectional returns, and evaluate the out-of-sample performance and economic significance of multi-factor volatility as compared to existing benchmarks.

Although financial theories do not specify the number and the identity of volatility risk factors, the prominent Fama–French (FF) three factors provide a natural guidance for our study on multi-factor volatility. This is motivated by some empirical evidence that the FF size and value factor are proxies for state variables that are linked to fundamental risk in the economy (Liew and Vassalou, 2000; Vassalou, 2003). We therefore investigate whether or not the volatility of the size and value factor captures the macroeconomic uncertainty risk by examining its impact on cross-sectional returns, its out-of-sample performance, and economic significance.

To accomplish this, we develop a two-stage multi-factor volatility model using a dynamic Fama–French three-factor approach. The first stage allows us to simultaneously estimate conditional means (as an AR process) and conditional variances (as a GARCH process) of the market, size, and value factor. The second stage estimates variance risk premia of the three factors and tests the cross-sectional pricing restriction. Non-arbitrage requires the existence of significant variance risk premia that drive pricing errors insignificant in the cross-section. In addition, we conduct out-of-sample predictive regressions and asset allocation tests to ensure that multi-factor volatility is non-elusive and carries a significant economic value.

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[1] For time-series studies, Ghysels et al. (2005), Guo and Whitelaw (2006) and Ludvigson and Ng (2007) find that aggregate volatility is positively related to market expected returns, whereas Glosten et al. (1993), Brandt and Kang (2004) and Christensen et al. (2010) document a negative relationship. In the cross-section of stocks, Ang et al. (2006) and Adrian and Rosenberg (2008) find a negative price of volatility risk, which is opposite to the prediction of the variance risk premium literature (Bollerslev et al., 2009; Drechsler and Yaron, 2011).

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Our results on the multi-factor volatility model are encouraging. Using the Fama–French 10×10 size and book-to-market sorted portfolios as the primary test portfolios,[2] we find that sensitivities to the conditional variances of the three factors exhibit wide dispersions across the 100 portfolios. Furthermore, the conditional variances of the size and value factor carry highly significant and positive risk premia, rendering the new model with a stronger asset-pricing performance than the competing models in terms of higher adjusted R-square, lower sum of squared errors, and lower pricing error statistics. To alleviate the potential model misspecification problem that might cause a spurious relationship, we test the pricing ability of multi-factor volatility using various asset-pricing models (i.e., the CAPM, ICAPM, and CCAPM), and still find significant results regardless of the models used. As for the out-of-sample forecasting performance, the multi-factor volatility model beats the existing benchmarks with a wide margin. For example, the model produces lower forecasting errors for at least 89 out of the 100 portfolios in the forecast of one-period-ahead expected returns. This suggests that its superior pricing power is not caused by adding some random factors; instead, multi-factor volatility is an incremental source of pervasive risk factors that should be priced both in sample and out of sample. To gauge its economic significance, our asset allocation results show that the multi-factor volatility model outperforms other models by about 1–3% annualized certainty equivalent returns (CER) in most cases, and the CER gap becomes even larger as investors exhibit more risk aversion. Our model also yields significantly higher Sharpe ratios than most benchmark models. As for practical implications, the asset-allocation results suggest that by accounting for multi-factor volatility into portfolio decisions, asset managers can significantly improve the risk-return trade-off of their portfolios.

The contribution of this paper is twofold. First, the paper sheds new light on the relationship between volatility risk and cross-sectional returns. In this literature, Ang et al. (2006) estimate a negative price of volatility risk of approximately -1% per annum.[3] The idea is that since innovations in volatility are higher during recessions, stocks that co-vary with volatility are stocks that pay off in bad states, and these should require a smaller risk premium. In a similar vein, Adrian and Rosenberg (2008) decompose market volatility into the short-run and long-run component and find that both components constitute negative volatility risk premia. On the other hand, theoretical work of Bollerslev et al. (2009) and Drechsler and Yaron (2011) predict a positive variance risk premium that compensates for macroeconomic uncertainty risk. Han and Zhou (2011) take a cross-sectional pricing approach and find that stocks with higher variance risk premium earn higher expected returns. These studies estimate a positive price of volatility risk using options on the aggregate market and individual stocks. It is infeasible, however, to estimate multi-factor volatility using options data. Using the cross-section of stock returns, rather than options data, allows us (1) to create portfolios of stocks that have different sensitivities to variations in multiple sources of volatility risk; (2) to control for a battery of other well-known risk-return effects. Our finding of positive prices of multi-factor volatility risk supports the prediction of the variance risk premium literature. We show that each factor volatility is positively correlated with variance risk premium. This implies that multi-factor volatility can serve as empirical proxy for variance risk premium, but with a stronger pricing ability. Indeed, in the presence of highly significant and positive premia for the volatility of the size and value factor, the risk premium for the market volatility becomes insignificant. This finding is robust to in-sample pricing, out-of-sample forecasting, alternative model and volatility specifications, and model test concerns raised by Lewellen et al. (2010).

Second, this paper contributes to the growing body of literature on out-of-sample return predictability by adding multi-factor volatility as a reliable set of predictors. In this strand of literature, Welch and Goyal (2008) and Simin (2008) argue that historical average returns forecast future returns better than predictive variables for both the market and individual stocks. Campbell and Thompson (2008) show that imposing meaningful restrictions on coefficients can largely improve the out-of-sample explanatory power of regressions. Rapach et al. (2010) recommend combining information from numerous economic variables to improve out-of-sample equity premium prediction. More recent studies propose technical indicators (Neely et al., 2014), time-varying coefficients (Dangl and Halling, 2012), sum-of-the-parts method (Ferreira and Santa-Clara, 2011), and conditioning on market state (Henkel et al., 2011; Zhu and Zhu, 2013; Zhu, forthcoming), among others, to predict stock market returns. In light of the literature, this paper argues that multi-factor volatility can significantly improve the out-of-sample return predictability. The economic gains of multi-factor volatility are evaluated by a dynamic asset-allocation approach of Tu (2010).

The remainder of our paper is organized as follows. Section 2 introduces the first stage of the multi-factor volatility model. Section 3 discusses the model estimation and estimated parameters. Section 4 conducts in-sample asset-pricing tests of the multi-factor volatility model as compared to the competing models. Section 5 performs out-of-sample forecasting tests of the multi-volatility model as compared to various benchmark models. Section 6 provides the asset allocation test to evaluate the economic value of the multi-factor volatility model. Section 7 concludes the paper with future research directions. The appendix includes details of the estimation procedure and results from Monte Carlo simulations.

2. The multi-factor volatility model

The Fama–French three-factor model (1993, 1996; FF3 hereafter) has been an empirical benchmark in the modern asset pricing literature. An open issue that has yet to be addressed in the literature is the relationship between the volatility of the Fama–French factors and expected stock returns. Fama and French (2012) explore the volatility of the market, size, and value premia of the three factors. Our paper formalizes this idea into conditional means and conditional variances of the three factors within a dynamic factor framework.

In the FF3 model, three factors originate from the 6 portfolios formed on size and book-to-market ratio (BTM). He et al. (2010) (HHL hereafter) use the Kalman filter to extract 3 latent dynamic factors from the 6 size and BTM portfolios by properly restricting factor loadings (betas). We adopt the HHL method and estimate the conditional means and conditional variances of the three factors simultaneously.

Denote the 6 size and BTM portfolios as SL, SM, SH, BL, BM, and BH. Let $\underline{R}_t = [\underline{R}_{SL,t}\ \underline{R}_{SM,t}\ \underline{R}_{SH,t}\ \underline{R}_{BL,t}\ \underline{R}_{BM,t}\ \underline{R}_{BH,t}]'$ be a vector of demeaned excess returns on the 6 portfolios in month t. The latent MKT, SIZE, and BTM dynamic factors are given by $D_t = (D_{MKT,t}\ D_{SIZE,t}\ D_{BTM,t})'$. With these notations, the dynamic factor model is represented in the state-space form, with the 6 observation or measurement equations given by:

[2] Given the extreme returns in some corner portfolios, the 100 portfolios pose a much greater challenge than the 25 portfolios (commonly used in the literature) in terms of in-sample pricing and out-of-sample forecasting. In addition, using 100 portfolios partially alleviates the model test problem raised by Lewellen et al. (2010).

[3] Another major finding of Ang et al. (2006) is that the idiosyncratic volatility relative to the Fama–French factors earns a significantly negative risk premium, which cannot be explained by the systematic market volatility. There is a contentious debate between Ang et al. (2009), Bali and Cakici (2008) and Fu (2009) with regard to the sign of idiosyncratic volatility risk premium. The asset-pricing role of idiosyncratic volatility is beyond the scope of this paper.