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Which continuous-time model is most appropriate for exchange rates? ☆

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ABSTRACT

This paper evaluates the most appropriate ways to model diffusion and jump features of high-frequency exchange rates in the presence of intraday periodicity in volatility. We show that periodic volatility distorts the size and power of conventional tests of Brownian motion, jumps and (in)finite activity. We propose a correction for periodicity that restores the properties of the test statistics. Empirically, the most plausible model for 1-min exchange rate data features Brownian motion and both finite activity and infinite activity jumps. Test rejection rates vary over time, however, indicating time variation in the data generating process. We discuss the implications of results for market microstructure and currency option pricing.

1. Introduction

What statistical model best describes the evolution of asset prices? Bachelier (1900) provided a very early attempt to answer this question when he modeled stock returns with "Brownian motion" (BM). More recently, the seminal work of Merton (1976) turned researchers' attention to modeling asset price jumps. The efficient markets hypothesis implies that asset returns exhibit limited predictability and that asset prices react rapidly to news surprises to prevent risk-adjusted profit opportunities. Thus, asset prices are likely to exhibit both continuous changes (diffusion) and discontinuous responses (jumps) to news. Decomposing returns into jumps and a diffusion with time-varying volatility is important because these two components imply different modeling and hedging strategies (Bollerslev and Todorov, 2011a,b). For example, although persistent time-varying diffusion volatility would help forecast future volatility, jumps might contain no predictive information about volatility (Neely, 1999; Andersen et al., 2007a). Therefore, one must jointly model jumps and volatility to better explain asset price dynamics.

Our paper investigates which continuous-time model best describes intraday exchange rate fluctuations. We address the following questions: Does an appropriate model need BM or infinite activity jumps? Are jumps present in the exchange rates? If so, do exchange rate jumps have finite or infinite activity?

The first class of models that we consider is the *Brownian SemiMartingale with Finite-Activity Jumps* (BSMFAJ) class. These models incorporate a diffusion—Brownian—component to capture the continuous variation of the price process and a jump component to account for price discontinuities.[1] These models exhibit *finite-activity* jump intensity—a finite number of jumps in any time interval. The compound Poisson process, which exhibits relatively rare and large jumps, is one example of this class. Andersen et al. (2007b) cite several authors who argue that the BSMFAJ class realistically models many asset prices.

BSMFAJ models may inappropriately restrict the jump dynamics of some asset prices, however. In particular, BSMFAJ

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[1] Semimartingale models are quite useful in financial economics and continuous-time finance because they rule out arbitrage opportunities (see Back, 1991).

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models allow only relatively rare and large jumps, despite evidence of many small jumps in equities (Aït-Sahalia and Jacod, 2011; Lee and Hannig, 2010). Therefore, Madan et al. (1998) and Carr et al. (2002) have introduced a new class of models with infinite-activity Lévy processes for jumps: the *Brownian SemiMartingale with Infinite-Activity Jumps* (BSMIAJ) class (Cont and Tankov, 2004).[2] The diffusion component of a BSMIAJ model captures the continuous price variation; and the jump component captures both rare and large jumps—potentially caused by important macro news (Dungey and Hvozdyk, 2012; Dungey et al., 2009; Lahaye et al., 2011)—as well as smaller, more frequent discontinuities that create risk for high-frequency trading strategies (Aït-Sahalia and Jacod, 2012). Jumps can arrive *infinitely* often in BSMIAJ models, which might better describe the data than the combination of BM and Poisson jumps (see Geman, 2002; Li et al., 2012, 2008).[3]

The choice of the most appropriate model has important implications for empirical asset pricing and equity research. For instance, a *Brownian SemiMartingale* (BSM) model—with only continuous components—generates different risk measures (and premia) than BSMFAJ or BSMIAJ models (Bollerslev and Todorov, 2011a,b; Drechsler and Yaron, 2011). Consumption-based asset pricing models (C-CAPM) rely on the continuous part of the returns and hence characterize only the diffusion risk (see Back, 1991).[4] As Maheu et al. (2013) show, jumps significantly contribute to the equity risk premium.

Few papers have investigated the best continuous-time model for exchange rates. Todorov and Tauchen (2010) and Cont and Mancini (2011) are most closely related to our paper; these works have also studied the characteristics of foreign-exchange-data-generating processes, focusing on jump activity and jump variation, respectively. Specifically, both papers use the Blumenthal-Getoor index to estimate jump activity and find that BM is present in foreign exchange data. We confirm this with power variation measures. We also confirm Cont and Mancini (2011)'s finding of finite jump intensity.[5]

Our work extends and complements these studies. Methodologically, we explicitly account for the effect of intraday periodicity in volatility on the flexible testing methodology of Aït-Sahalia and Jacod (2012). Our simulations show that intraday periodicity in volatility can badly distort the size and power of ASJ tests for Brownian motion, jumps, no-jumps, finite and infinite activity jumps. We propose a correction that removes the intraday periodicity in volatility and restores the desired properties of the tests.

We then apply these corrected tests to characterize the properties of a 10-year sample of 1-min data on three major exchange rates. Unlike the (jump) index estimation approaches of Todorov and Tauchen (2010) and Cont and Mancini (2011), we test the presence of each potential component separately (i.e., BM, jumps, and jump activity). These modified tests and long span of 1-min data yield new insights into the foreign-exchange-data-generating process. For example, Todorov and Tauchen (2010) conclude that the Brownian-plus-Poisson-jumps model might be misspecified, although they stress that this inference depends on the sampling frequency and testing technique. We show that a combination of Brownian motion, finite activity jumps and infinite activity jumps better characterize 1-min foreign exchange data than simpler, two-component models. Importantly, we also show that the test rejection rates display significant time variation, indicating that the most appropriate model of the data has probably changed over time, with more rejections of jumps and finite activity jumps. The long span of data was crucial to examine the previously neglected subject of time variation in the data generating process.

We organize our paper as follows: Section 2 presents the methodology. Section 3 describes the exchange rate data and extends the base technique to account for intraday periodicity. In Section 4, we conduct simulations to assess the finite sample properties of our testing procedures. Section 5 reports the empirical results and discusses the implications. Section 6 concludes.

2. The base methodology

2.1. Theoretical background

In line with previous literature, we assume that the log-price $X(t)$ follows a Brownian semimartingale with jumps such that

$$dX(t) = \underbrace{\mu(t)dt}_{\text{drift}} + \underbrace{\sigma(t)dW(t)}_{\text{continuous component}} + \text{JUMPS}(t), \quad t \geqslant 0, \tag{1}$$

where $dX(t)$ denotes the logarithmic price increment, $\mu(t)$ is a continuous, locally bounded, variation process, $\sigma(t)$ is a strictly positive and càdlàg (right-continuous with left limits) stochastic volatility process, and $W(t)$ is a standard BM. The JUMPS component of (1) potentially represents both finite- and infinite-activity jumps. That is,

$$\text{JUMPS(t)} := \underbrace{\kappa(t)dq(t)}_{\text{finite activity}} + \underbrace{h(t)dL(t)}_{\text{infinite activity}}, \tag{2}$$

where $q(t)$ denotes a counting process (e.g., Poisson process), $L(t)$ is a pure Lévy jump process (e.g., a Cauchy process), and $\kappa(t)$ and $h(t)$ denote the jump sizes of the counting and Lévy processes, respectively.

In the presence of noise, the true value of the log price is $X(t)$ but in the data we observe $Z(t)$. That is,

$$Z(t) = X(t) + e(t),$$

where $e(t)$ is the additive noise term as in Aït-Sahalia et al. (2012).

We assume that the log-price process (either $X(t)$ or $Z(t)$) is observed at discrete points in time. The continuously compounded ith intraday return of a trading day t is therefore given by $r_{t,i} \equiv X(t+i\Delta) - X(t+(i-1)\Delta)$ or $r_{t,i} \equiv Z(t+i\Delta) - Z(t+(i-1)\Delta)$, with $i = 1,\dots,M$ and trading days $t = 1,\dots,T$. Let $M \equiv \lfloor 1/\Delta \rfloor$ denote the number of intraday observations over the day; $\Delta = 1/M$ is the time between consecutive observations, the inverse of the observation frequency. In the absence noise and of the JUMPS component, (1) is a BSM model and therefore the realized variance, i.e. $RV_t = \sum_{i=1}^{M} r_{t,i}^2$, is a consistent estimator (when $\Delta \to 0$) of the integrated variance

$$IV_t \equiv \int_{t-1}^{t} \sigma^2(s)ds. \tag{3}$$

Some of the test statistics used in this paper are functions of "truncated power variations", a class of statistics that depend on three parameters: p, the power exponent, u, the truncation parameter, and k, the sampling frequency parameter. Aït-Sahalia and Jacod (2010) show that one can infer the properties of the data generating process from the probability limits of truncated power

[2] The literature studying infinite-activity Lévy processes is young but growing rapidly. Examples include the works of Aït-Sahalia and Jacod (2009a,b, 2010, 2012), Todorov and Tauchen (2006), Aït-Sahalia (2004), Carr and Wu (2003, 2004, 2007), Lee and Hannig (2010), Huang and Wu (2004), Carr and Madan (1998), and Bakshi et al. (2008), among others.

[3] It is worth noting that BSMIAJ models can exclude the finite-activity jump component so that only the infinite-activity jump component represents the price discontinuities.

[4] See also Lustig and Verdelhan (2007), who present empirical evidence on the link between aggregate consumption growth and exchange rate dynamics.

[5] See Bates (1996b), who propose a parametric model that captures stochastic volatility and jumps of deutsche mark options.